Tuning Time Delays to Improve the Performance of a Steering Controller

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Abstract. Time delays and lags in control loops can cause instability and pose significant challenges to engineers. This study investigates a steering controller using the dynamic bicycle model, where the steering system dynamics are approximated by a steering lag. A higher-level controller calculates the desired steering angle based on the vehicle's lateral position and yaw angle by considering various time delays related to these states. Stability charts are plotted for delay combinations, and the most stable gain setups for the feedback controller are determined. The results indicate that an appropriate increase in one of the time delays expands the stable domain of control gains for the vehicle system, and it enhances the performance of the vehicle controller.

 ${\bf Keywords:}\ time delay, steering control, stability analysis, feedback control$

1 Introduction

In the past twenty years, autonomous vehicle (AV) research has seen incredible progress. One area that is crucial for making these vehicles safe and stable is active steering control [1–3]. Even though control systems have made essential progress, surprisingly little attention has been given to studying the effects of time delays within them, despite the fact that time delays persist as a critical issue.

Recent research often focuses on automated platforms with complex electric circuits and advanced actuators, which cause large time delays and lags in the control system. However, most of the studies on path-following controllers [4, 5] neglect the impact of the sensor and communication delays and steering dynamics, although these may lead to degraded system performance, such as steering oscillations and instability [6, 7]. One significant challenge for contemporary AV systems is signal congestion. These systems are increasingly complex and face difficulties in assimilating large volumes of data. Variations in sensor configurations and estimation methods at the upper controller levels result in differences

in feedback delays related to state variables [8]. Steering lag is another critical factor [9] representing the time it takes for the tire to rotate and the tire contact patch to develop the slip angle necessary for generating lateral tire force. This lag is influenced by hardware capabilities and the design of the lower level control system. Hence, ignoring these two factors can potentially cause unforeseen effects on system behavior.

Therefore, an in-depth investigation into the effects of time delays in upperlevel controllers and steering lag in lower-level controllers is essential for developing robust and safe control systems for AV. In this study, the stability of a vehicle steering controller is analyzed considering time delays and steering lag. The stability domains of the control system and the most stable control gain setup can be determined under multiple time delay combinations. We can improve the performance of the control system via proper tuning of the delays.

2 Vehicle dynamics and control design

The lateral dynamics of vehicles are commonly studied using the well-known bicycle model, which assumes a constant longitudinal speed V_x (see Fig. 1). This model is widely used due to its simplicity and effectiveness in capturing vehicle behavior.

In this case, the planar bicycle model for the vehicle is introduced in the form

$$m(\dot{V}_y + V_x \dot{\psi}) = F_{\mathrm{F}y} + F_{\mathrm{R}y} \,, \quad J_{\mathrm{G}} \ddot{\psi} = a F_{\mathrm{F}y} - b F_{\mathrm{R}y} \,, \tag{1}$$

where V_y is the lateral speed, ψ is the yaw angle and δ is the steering angle. F_{Fy} and F_{Ry} are the lateral forces of the front and rear tires, respectively. The overall mass of the vehicle is m; J_G is the mass moment of inertia of the vehicle. Parameters a and b refer to the distances of the front and rear axles to the center of gravity G, respectively.

In case of small tire deformations, the linearized brush tire model [11] provides:

$$F_{\rm Fy} = -2C_{\rm F}\alpha_{\rm F}, \quad F_{\rm Ry} = -2C_{\rm R}\alpha_{\rm R}, \qquad (2)$$

where $C_{\rm F}$ and $C_{\rm R}$ are the so-called cornering stiffnesses of the front and rear tires, respectively. The tire slip angles are defined as follows:

$$\alpha_{\rm F} = \arctan \frac{V_y + a\dot{\psi}}{V_x} - \delta, \quad \alpha_{\rm R} = \arctan \frac{V_y - b\dot{\psi}}{V_x}.$$
(3)

3 Hierarchical steering control strategy

In this study, we consider that the desired path of the vehicle is along the X-axis. Namely, the lateral error of the vehicle is the position $Y_{\rm G}$ of the vehicle's center of gravity, while the angular error is equal to the yaw angle ψ . To accomplish the vehicle path-following, a hierarchical steering control strategy is constructed.



Fig. 1: Representation of planar bicycle model

The upper-layer control law for calculating the desired steering angle δ_d is designed to accommodate variations in feedback delays for lateral position and yaw angle. The control law is based on a linear state feedback:

$$\delta_{\rm d}(t) = -P_y Y_{\rm G}(t - \tau_y) - P_\psi \psi(t - \tau_\psi), \qquad (4)$$

where P_y and P_{ψ} are the feedback control gains. The time delays corresponding to the different signals are τ_y and τ_{ψ} .

In order to achieve the desired steering angle in the lower-level controller, a simplified model of the steering system is used with the first-order differential equation

$$\tau_{\rm s}\delta(t) = \delta_{\rm d}(t) - \delta(t),\tag{5}$$

where the steering lag $\tau_{\rm s}$ describes the latency of the steering. This lag is set to $\tau_{\rm s}=0.1$ s in the study [9]. $\delta(t)$ is the real steering angle of the front wheel, while $\delta_{\rm d}(t)$ refers to the desired steering angle.

Symbol (unit)	Description	Value
m (kg)	The total mass of the vehicle	1435
$J_z (\mathrm{kg} \cdot \mathrm{m}^2)$	Yaw moment of inertia	2340
L (m)	Wheelbase	2.5
a, b (m)	The distance from G to front, rear axle	1.2, 1.3
$C_{\rm F}~({\rm N/rad})$	Cornering stiffness of front wheel	$6 \cdot 10^4$
$C_{\rm R}$ (N/rad)	Cornering stiffness of rear wheel	$6 \cdot 10^4$
$V_x \ (\rm km/h)$	Longitudinal velocity	60

Table 1: Vehicle parameters.

4 Stability analysis

By means of the D-separation and the semi-discretization methods (see [12]), stability charts are constructed in the plane (P_y, P_{ψ}) of the higher level control 4 Jialin et al.

gains while all the other parameters of the system are fixed as shown in Table 1. The control gain setup, for which the system has the most stable configuration (i.e., when the largest absolute value of the characteristic multipliers of the semidiscretized system is minimal), can also be determined. This setup varies as the parameters of the system are changed, like in case of the variation of the time delays.

The state vector \mathbf{x} of the vehicle system is defined as $\mathbf{x} = [Y_{\rm G} \ \psi \ V_y \ \dot{\psi} \ \delta]^{\rm T}$. From Eq.(1)-(5), the linear state space model can be obtained:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t), \qquad (6)$$

where $\mathbf{A} \in \mathbb{R}^{5 \times 5}$, $\mathbf{B} \in \mathbb{R}^{5 \times 1}$. They are listed as follows:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{2(C_{\rm F}+C_{\rm R})}{m} & -\frac{2(C_{\rm F}+C_{\rm R})}{mV_x} & -\frac{2(C_{\rm F}a-C_{\rm R}b)}{mV_x} & \frac{2C_{\rm F}}{m} \\ 0 & \frac{2(C_{\rm F}a-C_{\rm R}b)}{J_z} & -\frac{2(C_{\rm F}a-C_{\rm R}b)}{J_zV_x} & -\frac{2(C_{\rm F}a^2+C_{\rm R}b^2)}{J_zV_x} & \frac{2C_{\rm F}a}{J_z} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{\rm s}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\tau_{\rm s}} \end{bmatrix}, \quad (7)$$

and the input u is the desired steering angle $\delta_{\rm d}$ that is given by the proportional controller

$$u(t) = \mathbf{K}_y \mathbf{x}(t - \tau_y) + \mathbf{K}_{\psi} \mathbf{x}(t - \tau_{\psi}), \qquad (8)$$

where the row vectors $\mathbf{K}_{y} = [-P_{y} \ 0 \ 0 \ 0 \ 0]$ and $\mathbf{K}_{\psi} = [0 \ -P_{\psi} \ 0 \ 0 \ 0]$.



Fig. 2: Stability charts in the plane of control gains P_y and P_{ψ} for different combinations of time delays.

Figure 2 shows the stable regions for the control gains P_y and P_{ψ} in case of several varying feedback delay combinations. The black dashed line outlines

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the stability domain, with darker gray shades indicating faster decaying signals, that is, indicating improved stability. The green star marks the optimal point for the fastest system decay, showing that maximum control gains do not yield the fastest decay. Increasing τ_{ψ} significantly reduces the stable region, while increasing τ_{y} primarily alters the shape of the stability domain. Some control gain combinations become stable as τ_u increases. In Figure 3(a), the blue domain represents the fastest decay for the combination of the time delays where the modulus of the largest characteristic multiplier is the smallest (see the colorbar to the right of the stability chart). To verify the effectiveness of adjusting a specific time delay combination to enhance the system's control performance, numerical simulations are conducted. The simulation duration is set to 10s, and the initial values are $\mathbf{x}_0 = [3.50000]^{\mathrm{T}}$. Simulations are carried out for two combinations of time delays: A ($\tau_y = 0.2 \,\mathrm{s}$; $\tau_\psi = 0.1 \,\mathrm{s}$) and B ($\tau_y = 0.4 \,\mathrm{s}$; $\tau_{\psi} = 0.1 \,\mathrm{s}$). Figure 3(b) illustrates the simulation results for the lateral position of the system over time. Combination B exhibits a faster system response compared to Combination A, reaching a stable state earlier. Hence, adjusting a specific time delay combination for the system under the most stable (optimal) gain combinations can enhance the control performance.



Fig. 3: The effect of different time delay combinations on the linear stability: (a) the value of the largest characteristic multiplier; (b) simulations for different time delay combinations.

5 Conclusion

In summary, the presented analysis identifies the control gain setup that yields the most robust parameter configurations against initial state perturbations by adjusting the time delays of the state signals processing. It is counter-intuitive that the optimal combinations of time delays are found with increasing one of the delays. These findings challenge the conventional wisdom that time delays 6 Jialin et al.

tend to destabilize dynamical systems: certain scenarios may benefit from larger time delays and these scenarios are also relevant in practical applications like the control of AV. Further investigation of complex hierarchical vehicle control systems with various delays is the task of future research.

6 Acknowledgements

The research reported in this paper was supported by the National Key Research and Development Program of the Ministry of Science and Technology of China under grant no. 2021YFE0116600, the Key Support Plan for Foreign Experts under grant no. zcgj2022017L, and the National Research, Development and Innovation Office of Hungary under grant no. 2020-1.2.4-TET-IPARI-2021-00012. D.T. was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, J.L. was supported by the China Scholarship Council. I.V was supported by the Rosztoczy Foundation.

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