Dynamics and synchronization of time-varying networks with coupling delays

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Interaction delays are ubiquitous in many real-life networks: it takes time for information to travel in communication networks, or between coupled optical elements. In the brain a coupling delay between interacting neurons arises from the conduction time of an electric signal along the axon. Here, we study the effect of a topology that changes over time in such delay-coupled networks. Network fluctuations are essential features of, for instance, interacting neurons, where synaptic plasticity continuously changes the topology, or networks modeling social interactions.

We concentrate on the synchronization properties of chaotic maps coupled with an interaction delay T_{d}. The coupling topology fluctuates between an ensemble of directed small-world networks, while keeping the mean degree constant. The network fluctuations are random, and not adaptive, i.e., the network evolution is not linked to the state. The dynamics is characterized by three timescales: the internal time scale of the node dynamics T_{in}, the connection delay along the links T_{d}, and the timescale of the network fluctuations T_{n}. When the network fluctuations are faster than the coupling delay and the internal time scale ($T_{\mathrm{n}} \ll T_{\mathrm{in}}, T_{\mathrm{d}}$) the synchronized state can be stabilized by the fluctuations: synchronization can be stable even if most or all temporary network topologies are unstable. As the network time scale T_{n} increases, the synchronized state becomes unstable when both time scales collide ($T_{\mathrm{n}} \approx T_{\mathrm{d}}$). Synchronization is more probable as the network time scale increases further. However, in the slow network regime ($T_{\mathrm{n}} \gg T_{\mathrm{d}} \gg T_{\mathrm{in}}$) we find that the long-term dynamics is desynchronized whenever the probability of reaching a non-synchronizing network is finite. Indeed, if the network acquires a desynchronizing configuration, it evolves sufficiently far away from the synchronized state, and the probability that subsequently sampled synchronizing networks take the chaotic maps back to synchronization manifold is negligible.

We complement these results with an analytical theory in the linearized limit: we develop Master Stability Function approach for time-varying networks with delayed connections and express the effective connectivity as a function of the three time scales T_{in}, T_{n} and T_{d}. Two limit cases allow an interpretation in terms of an effective network: When the network fluctuations are much faster than the internal time scale and the coupling delay ($T_{\mathrm{n}} \ll T_{\mathrm{in}}, T_{\mathrm{d}}$), the effective network topology is the average over the different topologies. When coupling delay and network fluctuation time scales collide ($T_{\mathrm{in}} \ll T_{\mathrm{n}} = T_{\mathrm{d}}$), the effective topology is the geometric mean over the different topologies (adjacency matrices).