On Basic Modeling of the Dynamics of Axles Rolling Process ⋆

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Abstract: In this work, a simple formulation of the axles rolling process is presented. This work only considers a simple representation of the elastic-plastic deformation subjected to regeneration that arises during the rolling operation. Simple two degrees of freedom mechanical model is introduced with an ideal constant control force. The deformation force is considered simplely according to the literature including elastic and elastic-plastic regions on the empirical deformation characteristics. In order to understand the process, steady rolling is extended to the real axles rolling process with constant feed, where overlapping can occur. The equations are simplified and time domain simulations are performed including this simple consideration of the elastic-plastic relay.

Keywords: rolling, delay, elastic, plastic, axles, dynamics

1. INTRODUCTION

Axles rolling process is an important cold forming process in the train industry. Almost all cases, train shafts have to be rolled to increase surface hardness by favourable residual compressive stress field, and to eliminate microcracks originated from previous turning processes (Carboni (2012); Mancini et al. (2006)). This removing of the microcracks can increase the lifetime of train shafts drastically, which is of a key importance in avoiding serious train accidents reported by Bracciali (2016).

This cold forming process is a fairly slow process and it requires heavy machinery due to the relatively high press force, that is usually granted by electro-hydraulic control system (Walters (1991)). With the rotating workpiece (shaft), one can expect similar regenerative effect that appears in cutting processes in Dombovari et al. (2010). Even though, in this case, the strict geometric accuracy is not that important, however, large amplitude vibrations can cause ripples on the surface, that is very much refused by quality standards.

Like any other forming process, rolling is also a quite complicated continuous mechanical process due to the simultaneous operation of elastic and plastic deformations. In the literature, sheet rolling process is quite extensively investigated by Kiutchi et al. (2000), Bruschi et al. (2014), and Lee et al. (2017); although it is quite a different process than axles rolling. In sheet rolling process, large portion of the workpiece is deformed, while in axles rolling the elastic foundation of the usually quite thin deformed layer is also has its importance.

That is why cylinder rolling on elastic-plastic material can come into the front. There are various attempts in the literature to model this operation on finite element basis (Jiang et al. (2002)). These models are quite specific, and due to their large sizes, usually they are not adequate for dynamic modeling. There are much more promising, although much complicated methods to describe the elastic-plastic deformation during cylinder rolling. These models have only a few parameters, however, those use plenty of assumptions related to the plastic zones along slip lines in Collins (1972).

We do not attempt to improve the inaccuracies of the formation models of cylinder rolling, described in the literature in Hambleton and Drescher (2009). By accepting their results, we derive the mathematical formalism of regenerative rolling processes that causes high amplitude vibrations.

The paper has the following structure. The simple rolling force characteristics is explained after Hambleton and Drescher (2009) and adjusted to the latter dynamic analysis. A two degrees of freedom (DoF) dynamic model is introduced with a constant control force to have the simplest possible model to describe an axles rolling process. A general model is introduced for steady rolling with no feed using one constant delay, that introduces the concept of plastic penetration function $\delta_{\mathrm{p}}(t)$ as an additional state of the system. This model is improved to operate with feed introducing multiple penetration functions. Lastly, the model is transformed into a multi-delayed dynamic model, that is more convenient to perform time domain simulations.

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2. SIMPLIFIED ROLLING FORCE MODEL

Simple plane strain model is considered here after the work of Hambleton and Drescher (2009). In that work an analytical formula is presented for narrow wheel penetration and rolling (see Fig. 1*a*). The paper of Hambleton and Drescher (2009) claims that, if the width b is relatively small compared to the radius of the wheel r, a simple power characteristics can be derived between the chord length h of the material contact and the penetration δ, as

$$\overline{h} = \sqrt{2\overline{\delta}}. \tag{1}$$

This formula takes into account the forward upheaval of the material by a slip line field as Collins (1972) did, and it differs from the experiments by increasing b. We use the following dimensionless parameters after Hambleton and Drescher (2009)

$$\overline{\delta} = \frac{\delta}{r}, \quad \overline{b} = \frac{b}{r}, \quad \overline{W} = \frac{W}{\kappa b r}, \quad \overline{H} = \frac{H}{\kappa b r}, \quad \overline{E} = \frac{E}{\kappa}, \tag{2}$$

where $\kappa = \frac{\sigma_0}{\sqrt{3}}$ is the shear yield stress governed by von Mises yield condition with the uniaxial yield stress σ_0. The rolling of material with Young modulus E produces deformation force Q with vertical W and horizontal components H.

According to Hambleton and Drescher (2009), by considering the force balance, a dimensionless average normal stress $\overline{q} = \frac{q}{\kappa}$ can be derived as

$$\overline{q} = 2 + \pi - 4 \arcsin \frac{\overline{h}}{2}. \tag{3}$$

This leads to the analytical formula of the dimensionless rolling force components (see (2)) as

$$\begin{aligned} \overline{W} &= \overline{q}\,\overline{h}\sqrt{1 - \frac{1}{4}\overline{h}^2}, \\ \overline{H} &= \frac{1}{2}\overline{q}\,\overline{h}^2. \end{aligned} \tag{4}$$

By accepting these formulae a nonlinear specific plastic force characteristics can be given as

$$\begin{aligned} f_{W,\mathrm{p}}(\delta) :=& \kappa r \overline{W} = \\ & \kappa r \left(2 + \pi - 4 \arcsin \sqrt{\frac{\delta}{2r}}\right) \sqrt{2\frac{\delta}{r}}\sqrt{1 - \frac{1}{2}\frac{\delta}{r}}, \\ f_{H,\mathrm{p}}(\delta) :=& \kappa r \overline{H} = \\ & \kappa r \left(2 + \pi - 4 \arcsin \sqrt{\frac{\delta}{2r}}\right) \frac{\delta}{r}. \end{aligned} \tag{5}$$

This specific force characteristics are depicted in Fig. 1*b*. We accept these simple formulae in their given form. Obviously, there are plenty of questions about the validity of these elastic-plastic models presented in Hambleton and Drescher (2009). Although, one could imagine the overall specific rolling force should vanish for no penetration, and should be a nonlinear function of the roller penetration δ, what is currently true for (5). But these formulae do not deal with the redistributed material along the side 'edges' of the roller, and do not describe the probably different rolling behaviour of multiple passes. The authors believe, there can be an improved model of (5), but that does not alter drastically the basic dynamic modeling issues of the rolling process presented below.

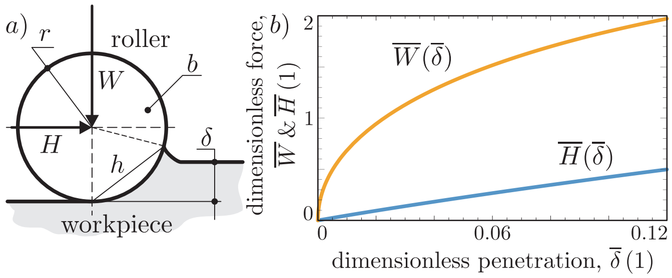

Fig. 1. *a*) shows the sketch of the rolling process, while *b*) presents the specific force based on Hambleton and Drescher (2009).

The specific force formulae (5) follow ones expectation to have much larger vertical force component than the one on the horizontal direction (Fig. 1*b*). However, the analytical expressions presented in (4) behave quite strange at zero penetration because the vertical force is initiated with an infinite tangent ($\frac{\partial \overline{W}}{\partial \overline{\delta}}(\overline{\delta} \to 0) \to \infty$), while the horizontal one with finite one with ($\frac{\partial \overline{H}}{\partial \overline{\delta}}(\overline{\delta} = 0) = 2 + \pi$). This is happening due to the completely different two mechanism that are dominating the vertical and horizontal directions. In the vertical direction rolling behaves more like a indentation, while in the horizontal direction the rubbing of the forward upheaval material is the influencing phenomenon.

In this very simple framework, the following description can be derived based on the elastic-plastic effect dominating the multi-passing rolling process. Post the yield condition the material is plastically deformed, then elastically released. Having the second loading, after the elastic uploading, the force is 'continued' exactly from the same level, from where the first plastic deformation had been released. This is very different than the operation of the cutting force characteristics, where in each cutting passes the same behaviour is happening over and over Dombovari and Stepan (2015).

This means, actually the material 'remembers' its deformation state, which can be modelled by using a coordinate transformation on the rolling force characteristics as presented in Fig. 2. In detail, the material undergoes plastic deformation from the intact state of surface A to the primary deformed state B. This penetration is measured with the value of plastic deformation δ_{p}. By releasing the surface from rolling pressure it recovers along its elastic stiffness K_W (W: vertical direction after Hambleton and Drescher (2009)) line to C. This released state is measured by the value of elastic-plastic deformation δ_{ep}. Note that, from now on, more deformation can only be achieved by using more force. That is, in the next round, if specific force on that surface segment is not increased the segment only undergoes elastic deformation only reaching D along

$$f_{W,\mathrm{e}}(\delta, \delta_{\mathrm{ep}}) = K_W(\delta - \delta_{\mathrm{ep}}). \tag{6}$$

Additional plastic deformation is only possible from D to E by pressing more the surface segment.

In order to ease the latter dynamic modelling, and since the actual rolling force characteristics is unknown, the rolling force is considered elastic until a limit penetration δ_{l} that can be calculated from the following equality

$$K_W\,\delta = f_{W,\mathrm{p}}(\delta) \quad \to \quad \delta_{\mathrm{l}}. \tag{7}$$

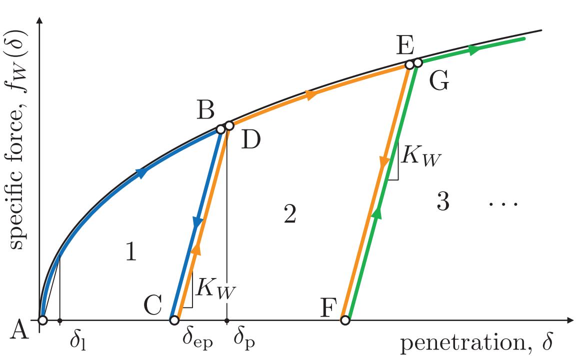

Fig. 2. The simple consideration of elastic-plastic effect during rolling by tracing plastic and elastic-plastic deformations δ_p and δ_{ep}, respectively.

Due to this simple description, an inconsistency can be felt with the very definition of the penetration δ, which was measured to the intact surface according to Hambleton and Drescher (2009). This penetration in the next pass would be much smaller, but this highly deformed elastic-plastic consideration of the rolling is not available yet in the literature. In this work, we point out the dynamic modeling problems, that can be later improved with the real rolling force characteristics considering multiple passes, which probably introduces different f_W & f_H characteristics for each number of passes.

3. DYNAMICS

In this section, we intend to model the behavior of the axles rolling process taking into account the simple rolling force behaviour in Fig. 2. The aim is to model the mechanics with an ideal control providing constant force $F_c = |\mathbf{F}_c| :=$ const. . It is necessary to know, in the standard, the desired pressing force is specified for a given rolling process not the value of the local plastic deformation. That means, the model lacks all electro-hydraulic components that keep this constant force $\mathbf{F}_c$ controlled. This actually results in, the model described below would not cover all types of self-excited vibration phenomena that cause poor rolled surface.

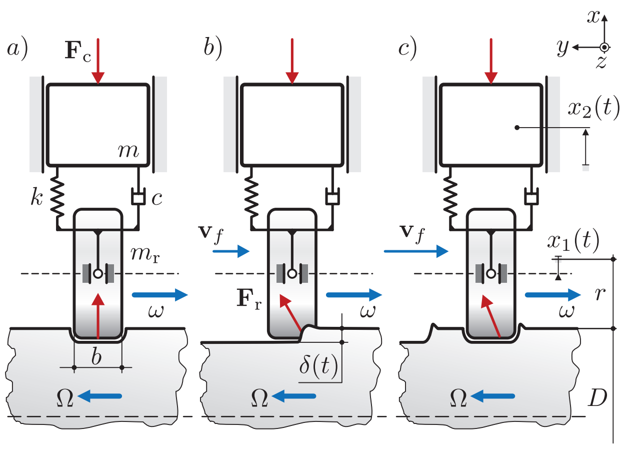

Fig. 3. The two DoF rolling model without feed *a*), with moderated feed ($\mathbf{v}_f$) *b*), and with extreme feed *c*). The assigned generalized coordinates (x_1, x_2) are presented in *c*).

Accepting the simple specific rolling force characteristic originated from Hambleton and Drescher (2009) at (5) and their elastic-plastic behaviour (see Fig. 2), the simplest two DoF mechanical model can be built with rigid roller and elastic-plastic workpiece (WP in Fig. 3). The dynamics of the machine is mimicked by one (dominant, not rigid) mode with a linear spring with stiffness k, and a linear viscous damper with damping b. The two DoF motion is described by two generalized coordinates x_1 and x_2 (see Fig. 3*c*), with which the roller centre (x_1) and the modal displacement of the dominant mode (x_2) are described.

One can build a model, where the roller rolls through the same material over and over (Fig. 3*a*), the one which actually performs real rolling process with moderate feed and overlapping (Fig. 3*b*). Lastly, the version with extremely high feed in such, the roller actually goes on a helical spiral without any overlapping (Fig. 2*c*). We describe the first as the learning simplest case, then we turn to the description of the real rolling process with moderate feed.

Constant uncontrolled force is pressing the mechanical system on the surface and producing the rolling force as the reaction. In the first model, there is no overlapping, the first pass performs the plastic deformation, then the elastic-plastic relay causes dynamic problems. In the second model, the feed drives the process described by the secondary 'feed' velocity $\mathbf{v}_f$ with overlapping, always introducing portions of the material, where the plastic deformation is dominant.

In both case, one can introduce the definition of the momentary penetration as

$$\delta(t) := -x_1(t), \tag{8}$$

which is strictly connected to the centre of the forceless rolling, when the roller just touches the workpiece.

In this sense, the specific rolling force depends on the momentary penetration $\delta(t)$ and it is integrated along β (see Fig. 4*a*) the contact length of the roller as

$$\mathbf{F}_r(\delta(t)) = \begin{bmatrix} F_W(\delta(t)) \\ F_f(\delta(t)) \\ -F_H(\delta(t)) \end{bmatrix} = \int_\beta \begin{bmatrix} f_W(\delta(t)) \\ f_f(\delta(t)) \\ -f_H(\delta(t)) \end{bmatrix} \mathrm{d}\beta. \tag{9}$$

In (9) f_W, f_H and f_f are the vertical, horizontal and feed directional specific force components. Note that. in this simple description the horizontal and feed direction does not effect the dynamics. Thus, during the process, the equilibrium state would be defined, when the x component of rolling force $\mathbf{F}_r$ is in balance with the control force $\mathbf{F}_c$. Other components are carried ideally by the frictionless guide constrains.

3.1 Equations of Motion

The equations of motion can be derived by e.g. Lagrange equation approach II by assuming the rolling force, which only depends on the momentary penetration $\delta(t)$ and does not on the penetration speed $\dot{\delta}(t)$.

In this manner a simple equations of motion can be derived as

$$\begin{aligned} m_r\ddot{x}_1(t) + c\dot{x}_1(t) - c\dot{x}_2(t) + kx_1(t) - kx_2(t) &= F_W(\delta(t)), \\ m\ddot{x}_2(t) - c\dot{x}_1(t) + c\dot{x}_2(t) - kx_1(t) + kx_2(t) &= -F_c. \end{aligned} \tag{10}$$

This two DoF model has a rigid body mode and a finite mode, as

$$\omega_{\mathrm{n},1} = 0 \quad \text{and} \quad \omega_{\mathrm{n},2} = \sqrt{k\frac{m+m_{\mathrm{r}}}{m\, m_{\mathrm{r}}}}. \tag{11}$$

Applying a real (not constant) control force F_{c}, which would be state dependent, the first natural frequency would be consolidated diverting from a rigid mode.

3.2 Zero feed simple rolling

In this modeling step, we attempt to see the modeling problems of the elastic-plastic relay in the rolling force. Note that, this does not a complete model of the real axles rolling process, but it is simple enough to show the main mathematics behind this cold forming process. In this simple case, the roller keeps passing the same groove over and over (see Fig. 3*a*).

In order to have a mathematical model, we need to introduce the definition of the momentary plastic deformation $\delta_{\mathrm{p}}(t)$, that stores actually the past motion of the roller x_1. Note that, due to the elastic release, the surface actually goes back to the defined elastic-plastic penetration as

$$\delta_{\mathrm{ep}}(\delta_{\mathrm{p}}) = \begin{cases} \dfrac{K_W\delta_{\mathrm{p}} - f_{W,\mathrm{p}}(\delta_{\mathrm{p}})}{K_W}, & \text{if}\, \delta_{\mathrm{p}} \geq \delta_{\mathrm{l}}, \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

This also means that, according to (6) $f_{W,\mathrm{e}}(\delta, \delta_{\mathrm{p}}) = f_{W,\mathrm{e}}(\delta, \delta_{\mathrm{ep}}(\delta_{\mathrm{p}}))$. Thus, the plastic deformation is stored in the following manner

$$\delta_{\mathrm{p}}(t) = \begin{cases} \delta_{\mathrm{p}}(t-\tau), & \text{if } \delta(t) \leq \delta_{\mathrm{p}}(t-\tau), \\ \delta(t), & \text{otherwise.} \end{cases} \tag{13}$$

This can drive the force equation by each moment checking the elastic-plastic-overpassing conditions, as

$$f_W(\delta(t), \delta_{\mathrm{p}}(t-\tau)) = \begin{cases} 0, & \text{if } \delta(t) \leq \delta_{\mathrm{ep}}(t-\tau), \\ f_{W,\mathrm{e}}(\delta(t), \delta_{\mathrm{p}}(t-\tau)), & \begin{array}{l}\text{if } \delta_{\mathrm{ep}}(t-\tau) < \delta(t) \\ \text{and}\, \delta(t) \leq \delta_{\mathrm{p}}(t-\tau),\end{array} \\ f_{W,\mathrm{p}}(\delta(t)), & \text{otherwise.} \end{cases} \tag{14}$$

The vertical force for ideal completely cylindrical roller for the case *a*) in Fig. 3 is $F_W(\delta(t)) = b f_W(\delta(t))$.

The piecewise definition in (12), (13) and (14) can be reformulated by Heaviside step function $H(x)$ as

$$\delta_{\mathrm{ep}}(\delta_{\mathrm{p}}) = H_1(\delta_{\mathrm{p}})\frac{K_W\delta_{\mathrm{p}} - f_{W,\mathrm{p}}(\delta_{\mathrm{p}})}{K_W}, \tag{15}$$

$$\delta_{\mathrm{p}} = (1 - H_2(\delta, \delta_{\mathrm{p},\tau}))\, \delta_{\mathrm{p},\tau} + H_2(\delta, \delta_{\mathrm{p},\tau})\delta, \tag{16}$$

$$\begin{aligned} &f_W(\delta, \delta_{\mathrm{p},\tau}) = H_3(\delta, \delta_{\mathrm{p},\tau})\cdot \\ &\quad ((1 - H_2(\delta, \delta_{\mathrm{p},\tau}))\, f_{W,\mathrm{e}}(\delta, \delta_{\mathrm{p},\tau}) + H_2\,(\delta, \delta_{\mathrm{p},\tau})\, f_{W,\mathrm{p}}(\delta))\,, \end{aligned} \tag{17}$$

where

$$H_1(\delta_{\mathrm{p}}) := H(\delta_{\mathrm{p}} - \delta_{\mathrm{l}}), \tag{18}$$

$$H_2(\delta, \delta_{\mathrm{p},\tau}) := H(\delta - \delta_{\mathrm{p},\tau}), \tag{19}$$

$$H_3(\delta, \delta_{\mathrm{p},\tau}) := H(\delta - \delta_{\mathrm{ep},\tau}). \tag{20}$$

Short term definition for the delayed terms are $\delta_{\mathrm{p},\tau}(t) := \delta_{\mathrm{p}}(t-\tau)$ and $\delta_{\mathrm{ep},\tau}(t) := \delta_{\mathrm{ep}}(t-\tau)$. Considering (8), the equations of motion can be written as a delayed differential algebraic equation (DDAE) by considering the plastic penetration δ_{p} as a state variable, and (16) as the related algebraic condition, thus

$$\begin{aligned} m_{\mathrm{r}}\ddot{x}_1(t) + c\dot{x}_1(t) - c\dot{x}_2(t) + kx_1(t) - kx_2(t) =& bH_3(-x_1(t), \delta_{\mathrm{p}}(t-\tau))\cdot \\ &((1 - H_2(-x_1(t), \delta_{\mathrm{p}}(t-\tau)))f_{W,\mathrm{e}}(-x_1(t), \delta_{\mathrm{p}}(t-\tau)) + H_2(-x_1(t), \delta_{\mathrm{p}}(t-\tau))f_{W,\mathrm{p}}(-x_1(t))), \\ m\ddot{x}_2(t) - c\dot{x}_1(t) + c\dot{x}_2(t) - kx_1(t) + kx_2(t) =& -F_{\mathrm{c}}, \\ \delta_{\mathrm{p}}(t) =& (1 - H_2(-x_1(t), \delta_{\mathrm{p}}(t-\tau)))\, \delta_{\mathrm{p}}(t-\tau) - H_2(-x_1(t), \delta_{\mathrm{p}}(t-\tau))x_1(t). \end{aligned} \tag{21}$$

3.3 Simple axles rolling with overlapping

The overlapping is caused by the constant feed $f = |\mathbf{v}_f|\tau$. Consequently, in each revolution of the workpiece, there is always a primary region, where mainly plastic deformation, while in the subsequent regions elastic-plastic deformation are working.

The constant feed f defines the width of the regions, simplely $b_l = f$ for each region $l = 1, \ldots, N-1$, where $N = \lceil b/f \rceil$. The last region has the width as $b_N = b - \lfloor (b-\epsilon)/f \rfloor f$ (ϵ is a sufficiently small number). Since, the regions on the surface moving forward, due to the constant feed, the local plastic deformation has the following connection with the previous region step ahead

$$\delta_{\mathrm{p},l}(t) = \begin{cases} \delta_{\mathrm{p},l-1}(t-\tau), & \text{if } \delta(t) \leq \delta_{\mathrm{p},l-1}(t-\tau), \\ \delta(t), & \text{otherwise.} \end{cases} \tag{22}$$

where $\delta_{\mathrm{p},l}(t) = 0$ if $l < 1$.

The lth's region elastic-plastic deformation $\delta_{\mathrm{ep},l}$ from elastic release is defined formally in the same way as (13). However, the vertical specific force component needs redefinition similarly to (14), as

$$f_{W,l}(\delta(t), \delta_{\mathrm{p},l-1}(t-\tau)) = \begin{cases} 0, & \text{if } \delta(t) \leq \delta_{\mathrm{ep},l-1}(t-\tau), \\ f_{W,\mathrm{e}}(\delta(t), \delta_{\mathrm{p},l-1}(t-\tau)), & \begin{array}{l}\text{if } \delta_{\mathrm{ep},l-1}(t-\tau) < \delta(t) \\ \text{and}\, \delta(t) \leq \delta_{\mathrm{p},l-1}(t-\tau),\end{array} \\ f_{W,\mathrm{p}}(\delta(t)), & \text{otherwise.} \end{cases} \tag{23}$$

By considering corresponding redefinitions of the switching functions given in (18)-(20), the following equations of motions can be derived for the simple axles rolling process with constant feed similarly the one in (21)

$$
\begin{aligned}
m_{\mathrm{r}}\ddot{x}_1(t) + c\dot{x}_1(t) - c\dot{x}_2(t) + kx_1(t) - kx_2(t) &= \sum_{l=1}^{N} b_l H_3(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau)) \cdot \\
((1 - H_2(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau))) f_{W,\mathrm{e}}(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau)) &+ H_2(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau)) f_{W,\mathrm{p}}(-x_1(t))), \\
m\ddot{x}_2(t) - c\dot{x}_1(t) + c\dot{x}_2(t) - kx_1(t) + kx_2(t) &= -F_{\mathrm{c}}, \\
\delta_{\mathrm{p},1}(t) &= -H_2(-x_1(t), 0)x_1(t), \\
&\vdots \\
\delta_{\mathrm{p},l}(t) &= (1 - H_2(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau)))\delta_{\mathrm{p},l-1}(t-\tau) - H_2(-x_1(t), \delta_{\mathrm{p},l-1}(t-\tau))x_1(t), \\
&\vdots \\
\delta_{\mathrm{p},N}(t) &= (1 - H_2(-x_1(t), \delta_{\mathrm{p},N-1}(t-\tau)))\delta_{\mathrm{p},N-1}(t-\tau) - H_2(-x_1(t), \delta_{\mathrm{p},N-1}(t-\tau))x_1(t).
\end{aligned}
\tag{24}
$$

The equation presented in (24) is again DDAE with one constant delay τ and N pieces of algebraic conditions and additional states $\delta_{\mathrm{p},l}$ to trace the local rolled surface.

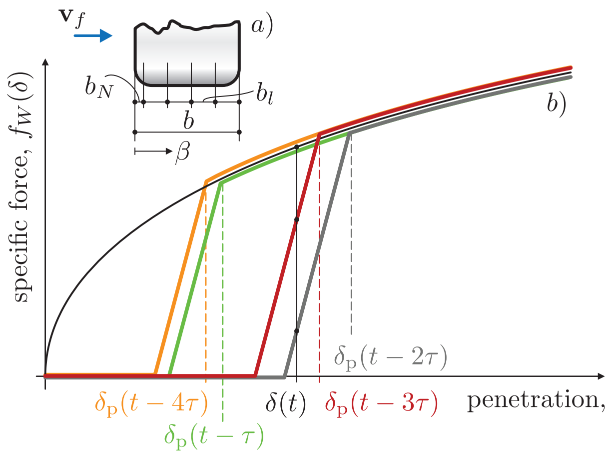

Fig. 4. In *a*) the parameters used for describing overlapping are presented, while *b*) shows the elastic-plastic force for different previous deformations.

4. TIME DOMAIN SIMULATIONS

The forms presented for steady rolling with no feed and the simple axles rolling process with feed in (21) and (24) are DDAE with one constant delay τ. These representations are much more suitable for solving boundary value problems, by considering different forms of plastic deformation as a state. An obvious smoothing is possible by smoothing out the Heaviside step functions directly with an adequate sigmoid function (di Bernardo et al. (2008)).

Avoiding these fairly complicated forms, one can introduce more delays naturally after the recursive definition presented in (13) and in (22). Since we used the steady rolling case only to understand the mechanics behind elastic-plastic deformation, we only deal with the simple axles rolling process with constant feed. In this manner, the recursive definition can be replaced by the following quite naturally

$$
\delta_{\mathrm{p},l}(t) = \max_{i=0}^{l-1} \delta(t - i\tau), \text{ for } l = 1, 2, \ldots, N. \tag{25}
$$

Consequently, all possible plastic deformations during the process are originated back to one of the delayed state, since (8). In Fig. 4*b*) the same train of thought is introduced as in (25) by considering all possible plastic and elastic-plastic states. Hence, one can realize always the minimum possible force is going to be realized connected possible plastic states (see $\delta_\mathrm{p}(t-2\tau)$ and $\delta_\mathrm{p}(t-4\tau)$).

For example, the current state $\delta(t)$ in Fig. 4*b* causes tiny elastic specific force part related to the previous remained plastic deformation $\delta_\mathrm{p}(t-2\tau)$. All other previous plastic deformations relate to a larger force, but those deformations are no longer active since (25). Consequently, on the lth portion the following force arises

$$
F_{W,l}(\delta(t)) = b_l \min_{i=0}^{l-1} f_{W,l-i}(\delta(t), \delta_{\mathrm{p},l-i-1}(t-(i+1)\tau)). \tag{26}
$$

Substituting consistently (25) into (26), summing the force portions, and finally considering the same two DoF dynamics introduced in (10) the following convenient form can be formulated

$$
\begin{aligned}
m_{\mathrm{r}}\ddot{x}_1(t) + c\dot{x}_1(t) - c\dot{x}_2(t) + kx_1(t) - kx_2(t) &= \sum_{l=1}^{N} b_l \min_{i=0}^{l-1} f_{W,l-i}\left(-x_1(t), \max_{j=0}^{l-i-2}(-x_1(t-(i+1+j)\tau))\right), \\
m\ddot{x}_2(t) - c\dot{x}_1(t) + c\dot{x}_2(t) - kx_1(t) + kx_2(t) &= -F_{\mathrm{c}}.
\end{aligned}
\tag{27}
$$

The form (27) does not contain any additional state variable related to the surface. Although, it is only convenient for rough time domain simulations, where 'min' and 'max' functions are evaluated momentarily fairly easily. In accurate time domain simulation a possible switch on the 'min' and 'max' functions has to be located with event detection. However, one can argue that many different effect actually influences this simple clear mathematical framework, and it is fair to say, the rough time domain simulation should be actually closer to the reality with its heuristic approach.

Sample simulations for zero initial condition are shown in Fig. 5. The simulations were performed, with the data presented in Tab. 1. Material properties were chosen heuristically only to perform some sample calculations. During the simulation internal damping has to be added at (6) vanishing high frequency vibrations. This is justified because during the process slip can introduce friction to dissipate some of the kinetic energy.

The simulations shows that large feed can stabilize the system, while more moderate feed can push the system to

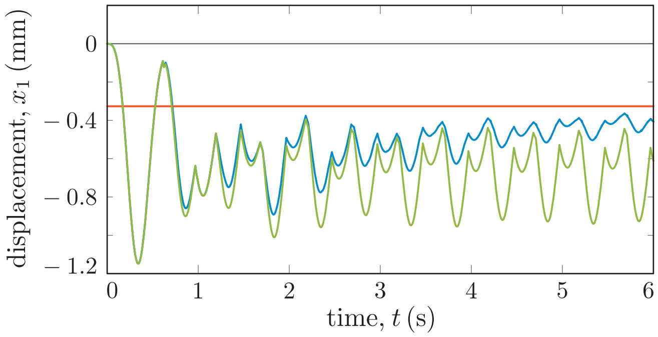

Fig. 5. Shows the equilibrium $\overline{x}_1$ (red line), where $F_c \equiv b f_W(-\overline{x}_1, -\overline{x}_1)$ & $\overline{x}_2 \equiv (k\overline{x}_1 - F_c)/k$. A stable ($f = 2\,\mathrm{mm/rev}$) and a saturated solution corresponding with an unstable ($f = 0.2\,\mathrm{mm/rev}$) equilibrium are depicted by blue and green.

σ_0 (MPa)	b (mm)	r (mm)	F_c (kN)	Ω (rpm)	ζ (1)
250	10	75	50	120	0.2
k (N/m)	m_r (kg)	m (kg)	K_W (GPa)	δ_l (mm)	ω_n (Hz)
275560	4.36	3525	20	0.195	40

Table 1. Data used in the simulations presented in Fig. 5.

a threshold motion. This is due to the different amount of regenerative overlapping operating in the rolling process.

5. CONCLUSIONS

The dynamic model of the axles rolling process was derived in this work. The goal of this research to reveal the dynamic modeling issues in this cold forming process. By introducing a simple elastic-plastic force characteristics the delayed differential algebraic equation (DDAE) form of the steady axles rolling process is introduced. The algebraic condition is the momentary plastic deformation defined as an additional state variable. By introducing the feed the surface regeneration is traced by the successive definition of the plastic deformations. In this manner, the DDAE form is extended, which is more adequate for solving boundary value problems.

This quite complicate DDAE form is simplified by introducing multiple constant delays. Consequently, the current plastic deformation is actually originated from all other previous state of the roller itself. The same idea is used for selecting the active elastic-plastic forming characteristics. In summary, a more simple form is derived, which is more adequate for time domain simulations.

By the introduced model, time domain simulation was performed that showed stable solution, and a solution keeping large amplitude considered as unstable in the engineering community.

This model is far to be used in industrial environment. It is more likely the used force model needs improvement considering hardening and the different upheaval material for multiple passes. Force control and the hydraulic drive that actually realizes the force control need to be modeled, which brings in different but important dynamic elements. Moreover, the entire machine has significantly larger dominant DoF's.

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