Stability of a spatially uniform steady state in two Turing systems coupled by a delay connection

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Amplitude death is a stabilization of equilibrium points embedded within oscillators coupled by connections. It is well known that amplitude death can occur if the frequencies of oscillations are non-identical, but never occurs if they are identical. On the other hand, it was reported that, if the connections include time delay, amplitude death can occur even in coupled identical oscillators [1].

Nonlinear spatiotemporal phenomena in reaction-diffusion systems have been widely studied in the field of nonlinear science. In particular, considerable attention has been paid to the research of Turing patterns in reaction-diffusion systems, because they reveal the fundamental mechanism for pattern generation in nature. The Turing patterns are caused by Turing instability. This instability is the phenomenon in reaction-diffusion systems that the diffusive term destabilizes the stable reaction dynamics.

We now focus on a pair of reaction-diffusion systems. Each system has the two-dimensional stable reaction dynamics and the diffusive term; a spatially uniform steady state exists in each system. Suppose that Turing instability occurs in each system: there exists at least one wavenumber at which the characteristic function describing stability of the steady state is unstable. Let us consider the stability of the spatially uniform steady state in a pair of reaction-diffusion systems coupled by the following two types of connections: a diffusive (i.e., no-delay) connection and a time-delay connection. It was analytically guaranteed that the diffusive connection never induce the stabilization of the steady state in coupled identical reaction-diffusion systems [2].

This report deals with the stability of the steady state with the time-delay connection: the characteristic function of the steady state in the coupled systems can be reduced to a product of two functions. We can analytically show that one of the functions has at least one positive root due to Turing instability. Therefore, it is guaranteed that the stabilization in coupled reaction-diffusion systems cannot be induced even by the time-delay connection. It should be noted that this is not similar to the result of time-delayed coupled oscillators. This fact might essentially stem from the odd-number property of time-delayed coupled oscillators [3].

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