Applications of Optimal Control Problems with Time Delays

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We consider optimal control problems with time delays in state and control variables. The control system is subject to control and state constraints. We discuss the necessary optimality conditions in the form of a Pontryagin type Minimum Principle which have been obtained only recently by R. Vinter and the author. We propose discretization and nonlinear programming methods to solve the time-delayed optimal control problem which extends the methods in [2]. The discretization approach furnishes also the Lagrange multipliers and adjoint variables of the control problem. This allows us to check the necessary optimality conditions with high accuracy. As a first application we discuss the optimal control of a Chemical Two-Tank Reactor (CSTR) with a delay in the state variables which is caused by the transport between the two tanks. We also impose terminal constraints and a state constraint on the temperature in the second tank. In the second application we study a delay differential model of the chemo-immuno-therapy of cancer with time delays in control and state variables [1]. We also impose a state constraint on the state variable representing the healthy cells. Further applications of delayed optimal control problems in biomedicine may be found in [3,4,5].

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