H_∞ optimal observer design for time-delay systems using SOS

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In recent years, there has been considerable progress in the development of asymptotic algorithms for control of systems with time delay. Simply put, an asymptotic algorithm is defined as a sequence of algorithms, indexed by some metric of complexity, each instance providing a sufficient condition, being of polynomial-time complexity, and where as the sequence progresses, the complexity and accuracy of the algorithms increase – presumably to some notion of necessity. Examples of asymptotic algorithms for stability analysis of time-delay systems include the Piecewise linear approach of Gu, the Wirtinger approach of Seuret et al. and the SOS approach.

The goal of this talk is to extend our recent success in the development of asymptotic algorithms for time-delay systems to the problem of H_∞-optimal state observer synthesis. An H_∞-optimal observer is a dynamical system which corrects its estimate of the state based on measurements of the physical system and minimizes the effect of disturbances to the physical system on the error between the estimate and the actual state.

When time-delay is included in the dynamics, the state of the system is a combination of the present state and the history of the state over the period of delay. For this reason, the estimator dynamics for a time-delay system include both an estimate of the current state and an estimate of the history of the state. In the ideal case, real-time measurement errors should then be used to correct both the estimate of the history and the estimate of the current state. Until the 1990's it was universally acknowledged that this was a natural framework for estimator design and that the resulting observer structure is a PDE and the observer gains are functions rather than matrices. However, the use of PDE observers for time-delay systems was abandoned in the mid-1990s when Ricatti and LMI methods made it possible to search for quadratic Lyapunov functions and matrix-valued feedback.

Unlike traditional LMI-based methods, asymptotic algorithms based on SOS optimize polynomial variables which can then be used to parameterize Lyapunov functions of the "complete quadratic" type known to be necessary and sufficient for stability of a time-delay system. Recently, this approach has been formalized within the semigroup framework by demonstrating that the SOS polynomial variables represent a parametrization of the positive linear operator variables.

The representation of control problems as optimization of operators is termed a Linear Operator Inequality (LOI) for the purpose of denoting the obvious connection with LMIs – wherein the variable is a matrix. This parallelism is not superficial, however, as many LMI results have direct analogues as solvable LOIs. While we do not belabor the point, the significance is that the LMI for H_∞-optimal observer synthesis can now be represented as an LOI for which we have asymptotic algorithms. As is seen in the numerical examples, the provable H_∞ gain of such observers is very often an order of magnitude less than existing results and furthermore, simulation shows that these predicted gains are tight, indicating the conditions have little if any conservatism.