# On control and anticipatory effects in applied mechanics

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There are numerous papers dealing with anticipatory systems [3], discretization and stability analysis [1], [2]. In applied mechanics controlled system may exhibit anticipatory properties. The system is called anticipatory, when its present state is determined by its future or its present states [2].

In classical dynamics Newton's second axiom presents us the equation of motion as

$$m\ddot{x} = F.$$

When $F$ depends on $x$, a position control is applied, which appears in positioning or balancing problems. By using conventional control theory proportional, derivative or integrative feedback may be applied. Consequently in position control besides its derivative or integral may appear as variable of the force.

In several engineering problems accelerometers are also used to present input for the control. By measuring the acceleration of some elements of a mechanical system the data can be used as input signal to the controller. Then a force or torque is generated and applied to the parts of the system to ensure a prescribed behavior. The equation of motion for such systems contains higher order derivatives and may cause difficulties in mathematical analysis.

In the simplest case (single degree-of-freedom, proportional control) a "naive" formulation leads to

$$\begin{aligned} \dot{x} &= v \\ m\dot{v} &= a_0 x + a_1 v + a_2 \dot{v}. \end{aligned} \tag{1}$$

When numerical methods are used, time discretization is necessary. Then anticipatory systems may appear. In the previous example the discrete time version reads

$$\begin{aligned} x(n+1) &= x(n) + v(n) \triangle t \\ v(n+1) &= v(n) + a_0 x(n) \triangle t + a_1 v(n) \triangle t + a_2 \left( v(n+1) - v(n) \right). \end{aligned} \tag{2}$$

In (2) the future state $(v(n+1))$ of the system depends on itself, which is a special type (incursive system) of anticipatory behavior [2]. For more sophisticated controls, with or without time delay and multi degrees-of-freedom we get more complicated equations. Such systems exhibit much stronger anticipatory properties. Moreover, equations like (1) may cause serious modeling problems in analytical mechanics.

Our paper aims to present a detailed study of such systems focusing attention on dynamic stability.

*Keywords:* applied mechanics; controlled systems; discretization; stability analysis

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