State predictive control by integral approximation for inverted pendulum with time delay

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It is well known that the state predictive control is effective in the design of the controller for the multivariable system with time delay in control. It was applied to the issue of finite spectrum assignment, LQ optimal regulator and H-infinity control. It has the state-feedback control and the distributed-delay control to integrate input history in the time delay period. It is necessary some approximations such as the rectangular, trapezoidal approximation $\{Z_{tr}(s)\}$, Simpson's rule and so on when the state predictive control is implemented to a real plant. Though the distributed-delay control is strictly proper, the approximation becomes no strictly proper because of the intrinsic high frequency mismatch.

For this problem, Mirkin transformed the distributed-delay by equivalent conversion and proposed an approximation method that gave an explicit low-pass filter to the outside (Mirkin, 2004). The high-frequency gain of the approximation error can reduce arbitrarily by the time constant of the low-path filter. Note that trapezoidal approximation will be used for the integral part. $\{Z_f(s)\}$

On the other hand, Qing-Chang Zhong expressed the divided integral interval as a convolution of the integrable function and the pulse, and proposed an approximation method by using the mean value of the matrix exponential function (Q-C Zhong, 2004). It is shown that the error becomes 0 in the low frequency band. However, an inverse matrix of the system matrix A is needed, therefore a pseudo-inverse matrix is necessary and some error is left generally. $\{Z_c(s)\}$

Though destabilization due to the approximation is a problem of the implementation to a real plant, most of the experimental example cannot be found. A stabilized approximation with little divided number is expected by limitation of hardware such as the memory. We applied the Mirkin approximation to the inverted pendulum, and confirmed that it was possible by the experiment in the little divided number enough than the trapezoidal approximation (Abe et al. 2010).

In this presentation, we apply the approximation method of Mirkin and Q-C Zhong for the rotary inverted pendulum with a flexible arm. Because the plant has the flexible arm, we should suppress the vibration of the arm and stabilize the pendulum by using LQR with frequency-shaped weights. By the trapezoidal and Mirkin approximation, a difference of the remarkable performance is not seen in experiments and analysis. In both cases, it is caused by increasing gains in a low frequency band. On the other hand, an effective result was provided with this plant because the gain of the low frequency band was suppressed by the Q-C Zhong method.

Mirkin approximation is effective in the high frequency band, however, the property of trapezoidal approximation is appeared in the low frequency band. Therefore, it can be expected that it is suppressed in both high and low frequency bands by using Q-C Zhong approximation instead of the part of trapezoidal approximation. $\{Z_{fc}(s)\}$ We are able to demonstrate that the mixed approximation can reduce the most divided number by experiments. We show a video by the presentation.

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\begin{aligned}
v(t) :=& \int_0^L \mathrm{e}^{A\beta} B u(t-\beta)\mathrm{d}\beta \\
Z(s) =& \frac{V(s)}{U(s)} = (sI-A)^{-1}\left(I-\mathrm{e}^{-(sI-A)L}\right)B \\
Z_f(s) =& \frac{1}{\tau s+1}\left\{\tau\left(I-\mathrm{e}^{-(sI-A)L}\right)B+(\tau A+I)Z_{tr}(s)\right\} \\
Z_c(s) =& \frac{1-\mathrm{e}^{-\frac{L}{N}s}}{s}\left(1-\mathrm{e}^{-A\frac{L}{N}}\right)\left(\frac{L}{N}A\right)^{-1}\sum_{k=1}^{N}\mathrm{e}^{-k\frac{L}{N}(sI-A)}B \\
Z_{fc}(s) =& \frac{1}{\tau s+1}\left\{\tau\left(I-\mathrm{e}^{-(sI-A)L}\right)B+(\tau A+I)Z_c(s)\right\}
\end{aligned}
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