Differential equations with gamma-Ricker type delayed feedback

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The gamma-Ricker map is a flexible function widely used in mathematical models to fit population data [3]. Its flexibility comes from the fact that it involves three parameters, which allows to model different forms of density dependence.

We consider the family of delay differential equations

$$x'(t) = -\alpha x(t) + \beta x^{\gamma}(t-\tau)e^{-\delta x(t-\tau)}, \tag{1}$$

where $f(x) = \beta x^{\gamma} e^{-\delta x}$ is the gamma-Ricker map. The five involved parameters are assumed to be positive.

Equation (1) with different values of γ has been used in different mathematical models. The limit case $\gamma = 0$ was proposed by Lasota and Wazewska in 1976 to model erythropoiesis (red blood cell production), while the general case $\gamma > 0$ was introduced later by Lasota [2] to model disturbed erythropoiesis; for $0 < \gamma < 1$, (1) is a generalization of the fundamental Solow's neoclassical growth model in economics [1]; when $\gamma = 1$, equation (1) is the famous Nicholson's blowflies equation; and for $\gamma > 1$, (1) has been proposed to study populations subject to Allee effects [4].

We present some new results on the global dynamics of (1) in the cases $0 < \gamma < 1$ and $\gamma > 1$, paying special attention to permanence and stability. Bifurcation diagrams using relevant model parameters show some interesting features, such as stability switches and extinction windows due to sudden collapses. In the framework of population dynamics, it is especially interesting to study the influence of γ, which can be considered as a cooperation parameter.

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