Dynamics of coupled Duffing oscillators with delayed feedback

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The typical nonlinear Duffing oscillator has been studied very often in the literature because of its interesting dynamics and possible applications [1]. Coupled oscillators without time delay have been widely studied as a mathematical model for understanding the collective behaviour of a wide variety of physical, chemical and biological problems. The coupling of a few nonlinear oscillators may often produce many new interesting phenomena, which in a case of a single oscillator cannot be observed. For coupling systems, in reality input signal from one oscillator reaches the others after a certain time delay due to the finite speed of the signal propagation. Time delay is inevitable in real structures. Effects of time delay are also very popular in the study of dynamical systems with many delay factors that appear in state variables and some of them appear in parameters. Sometimes, the time delay is introduced into the system on purpose to control its response.

Differential equations with time delay correspond to an infinite dimension in phase space. The most often, the systems described by autonomous delay differential equation (called sometimes difference-differential equation) is taken into account but the dynamics of non-autonomous systems have received less attention. However, some papers treat this problem with special concern, for instant where the first order approximation of multiple scales method is engaged to solve primary resonance problem [2, 3], but sometimes such approach is not sufficient to present system response accurately.

The Duffing oscillators are investigated in two aspects. First of all, how delay displacement feedback influences the primary resonance of the classical coupling Duffing oscillators in order to control the system and on the other hand, how the external harmonic force influences vibrations of the system with time delay which is essential in applications e.g. in a cutting process.

This paper presents results of analytical and numerical coupled Duffing oscillators described by differential equations with time delay. Analytical solution of the model near the primary resonances are obtained by using the method of multiple time scales. An influence of basic system parameters on the dynamics is determined analytically and then verified numerically.

It has been observed that the change of a gain of the delay signal shifts the resonance curve in frequency domain while external excitation changes the amplitude of the system. An increase of the amplitude of the external excitation causes an increase in the amplitude of the system oscillation, while it does not affect the occurrence of the unstable area. A change of the time delay value does not affect the position of the resonance curve, while it causes a change of stability and affects the amplitude of the system vibration. It was also found that by appropriate selection of the time delay one can change dynamics of the system from stable to unstable or vice versa.

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