Structured H_∞-control of infinite dimensional systems

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In this work we use a frequency-based H_∞-method to control infinite-dimensional LTI-systems $G(s)$ covering subclasses of systems with delay dynamics. After embedding $G(s)$ as usual in a plant $P(s)$ and setting up performance and robustness channels as a transfer function $T_{wz}(P, K)$, from w to z, we address the infinite-dimensional H_∞-optimization problem

$$\begin{array}{ll} \text{minimize} & \max\limits_{\omega\in[0,\infty]} \overline{\sigma}\left(T_{zw}\left(P(j\omega), K(j\omega)\right)\right) \\ \text{subject to} & K \text{ stabilizes } G \text{ exponentially} \\ & K \in \mathscr{K} \end{array} \tag{1}$$

where optimization is over a class $\mathscr{K}$ of structured finite rank control laws. Our strategy is to choose frequency samples $G(j\omega_\nu)$ of $G(s)$ in such a way that the solution $K^* \in \mathscr{K}$ of the approximate H_∞ program

$$\begin{array}{ll} \text{minimize} & \max\limits_{\nu=1,\dots,N} \overline{\sigma}\left(T_{zw}\left(P(j\omega_\nu), K(j\omega_\nu)\right)\right) \\ \text{subject to} & K \text{ stabilizes } G \text{ exponentially} \\ & K \in \mathscr{K} \end{array} \tag{2}$$

guarantees closed-loop stability for $G(s)$, and assures that the value of (2) differs only by a fixed tolerance ϑ from the true value of (1). Sampling in the frequency domain becomes necessary since the objective of (1) is semi-infinite, non-smooth, and non-convex, and not directly amenable to efficient computation.

The difficulty in program (1) is further aggravated by the fact that controllers $K \in \mathscr{K}$ have to be *structured*. Structured controllers are preferred by practitioners and include classics like PIDs, lead-lag-notch filters, reduced fixed-order controllers, etc. For systems given in state-space, we obtain the transfer function $G(s)$ directly from the infinite dimensional system. Our method is also suited for systems provided from start in frequency sampled form (2), or for systems given directly by their transfer function.

Our contribution is threefold and covers the following topics:

(a) How to sample the MIMO Nyquist test so that exponential stability in closed loop is guaranteed.

(b) How to sample the transfer function $G(s)$ so that the approximate value of (2) is within a fixed tolerance ϑ of the true value of (1).

(c) How to (locally) solve the non-smooth optimization program (2) algorithmically.

To address the stability issue (a) we implement an infinite-dimensional Nyquist test, which is effective as soon as stability of the closed-loop systems is *spectrum-determined*. Optimization (c) is based on the non-smooth trust-region method.

The proposed approach is validated through numerous testing cases, finite- and infinite -dimensional systems, PDEs as well as systems with delay dynamics.

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