On Bounded Control of A Class of Feedforward Nonlinear Time-Delay Systems ⋆

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Abstract: For the global stabilization of a family of feedforward nonlinear time-delay systems with linearized identical oscillators, a saturated feedback control is established based on a special canonical form for the considered system. The proposed control laws use not only the current states but also the delayed states for feedback, and, moreover, contain some free parameters. These advantages can help to improve the transient performance of the closed-loop system significantly. A numerical example is given to illustrate the effectiveness of the proposed approaches.

Keywords: Bounded controls; Feedforward nonlinear systems; Time-delay systems; Global stabilization; Nonlinear feedback.

1. INTRODUCTION

Delays and boundedness are ubiquitous in systems (Chen et al. (2014), Murguia et al. (2014), Wei et al. (2017), Zhou et al. (2013)). Ignoring time delays/boundedness in the design of control systems will degrade the system performances and may even lead to instability. For this reason, a subject of corresponding research activities have increased over the past two decades (Giannini et al. (2016), Liu et al. (2014), Marchand et al. (2005), Selivanov et al. (2016), Zhang et al. (2011)). Among the research activities, an important fundamental problem is the global stabilization of control systems with bounded (and delayed or not) controls (Kaliora et al. (2004), Mazenc et al. (2003), Mazenc et al. (2004), Yakoubi et al. (2007)). For instance, In Teel (1992), nonlinear state feedback laws were first proposed by Teel for the global stabilization of the multiple integrators without delay. Teel's pioneer work was later successfully extended to general ANCBC linear systems (Sussmann et al. (1994), Yang et al. (1997)), and even some nonlinear systems including feedforward nonlinear systems described by equations with an upper triangular structure (Mazenc et al. (2004), Ye (2014), Ye (2011)).

Motivated by Teel's forwarding design in Teel (1992), Mazenc et al. studied the problem for the global stabilization of nonlinear systems in feedforward form with bounded and delayed feedback in Mazenc et al. (2004). A class of nonlinear control laws consisting nested saturation functions was established and explicit expressions of bounded control laws were determined. Based on the transformed nonlinear system given by Mazenc et al. (2004), another nonlinear control law consisting cascade saturation functions was proposed in Ye et al. (2012) by Ye et al.. Later on, Ye investigated another kind of feedforward nonlinear systems with nominal dynamics being the cascade of multiple oscillators and multiple integrators in Ye (2014). The saturated delayed feedback was proposed for the global stabilization problem. However, different from the analysis in Teel (1992), because of the existing time delay in the input, the decoupling property is no longer valid in the recursive designs in Mazenc et al. (2004); Ye (2014); Ye et al. (2012), which makes the corresponding analysis more complicated. In order to overcome the shortage, recently, based on some special canonical forms which can keep the decoupling property in the designs, we proposed three globally stabilizing nonlinear control laws containing not only the current states but also the delayed states for global stabilization of a chains of integrators subject to input saturation and delay in Zhou et al. (2016). Later on, the methods in Zhou et al. (2016) have been extended in Zhou et al. (2018) and Yang et al. (unpublished) to continuous/discrete feedforward nonlinear systems whose nonlinearities contain not only the current states but also the delayed states.

In this paper, we consider the global stabilization of a family of feedforward nonlinear time-delay systems with linearized identical oscillators by bounded controls. Motivated by our recent results in Yang et al. (2017) and Zhou et al. (2018), a special state space description containing

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both the current and delayed state vectors of the considered system will be constructed. The transformation could help to maintain the decoupling property in the recursive design. Based on the special canonical form, a class of nonlinear control laws consisting cascade saturation functions will be proposed to solve the problem, and explicit conditions will also be proposed for the control laws to guarantee the global stability of the closed-loop systems. Different from our previous works in Zhou et al. (2018) where nonlinear time-delay systems with linearized multiple integrators were considered, the systems (see Eq.(1)) we consider here are more complicated to handle. For instance, in order to guarantee the saturation functions operate in linear region after a finite time, a deeper analysis than that in Zhou et al. (2018) will be presented. Compared with the results in Ye (2014), our proposed nonlinear control laws contain not only the current but also the delayed states information, which allows us to cancel all the other state components at every step of the recursive design and naturally leads to a more concise analysis than that in Ye (2014). Moreover, the design approach proposed in this paper can deal with feedforward nonlinear systems containing not only the current states but also the delayed states, which was considered in Ye (2014).

Notation: The notation used in this paper is standard. For two integers p and q with $p \leq q$, the symbol $\mathbf{I}[p,q]$ refers to the set $\{p, p+1, \ldots, q\}$. For a positive constant ε, $\sigma_\varepsilon(x) \triangleq \varepsilon \operatorname{sign}(x) \min\{|x/\varepsilon|, 1\}$ denotes the standard saturation function. The notation $|\cdot|$ refers to both the usual Euclidean norm for vectors and the induced 2-norm for matrices. Finally, for any constants a and b with $b \geq a$, we let $y_{[a,b]} = y(s), s \in [a,b]$ and $|y|_{[a,b]} \triangleq \sup_{s\in[a,b]} |y(s)|$.

2. PROBLEM FORMULATION

In this paper, we consider the following feedforward nonlinear system:

$$\begin{cases} \dot{x}_1(t) = A_\omega x_1(t) + Ax_2(t-h_2) + f_1, \\ \dot{x}_2(t) = A_\omega x_2(t) + Ax_3(t-h_3) + f_2, \\ \quad\vdots \\ \dot{x}_{p-1}(t) = A_\omega x_{p-1}(t) + Ax_p(t-h_p) + f_{p-1}, \\ \dot{x}_p(t) = A_\omega x_p(t) + bx_{p+1}(t-h_{p+1}) + f_p, \end{cases} \tag{1}$$

where $f_i = f_i((X_{i+1})_{[t-r,t]})$, and

$$A_\omega = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}, \; A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \; b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

in which $\omega \neq 0$, $p \geq 1$, $h_i, i \in \mathbf{I}[2, p+1]$, are non-negative numbers, r is a non-negative constant that can be unknown, $x = [x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_p^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{2p}$ with $x_i = [x_{i1}, x_{i2}]^{\mathrm{T}} \in \mathbf{R}^2, i \in \mathbf{I}[1,p]$, is the state vector, $x_{p+1} = u \in \mathbf{R}$ is the input, and $X_i = (x_i^{\mathrm{T}}, x_{i+1}^{\mathrm{T}}, \ldots, x_p^{\mathrm{T}}, x_{p+1})^{\mathrm{T}}$ for any $i \in \mathbf{I}[1, p+1]$. The functions $f_i = [f_{i1}, f_{i2}]^{\mathrm{T}} \in \mathbf{R}^2, i \in \mathbf{I}[1,p]$, are continuous and satisfy the following assumption.

Assumption 1. There exist positive scalars $\phi_i, i \in \mathbf{I}[1,p]$, such that

$$\left| f_i\left((X_{i+1})_{[t-r,t]}\right)\right| \leq \phi_i \left|X_{i+1}\right|^2_{[t-r,t]}, \tag{3}$$

whenever $|X_{i+1}|_{[t-r,t]} \leq 1$.

In this paper we aim to solve the following problem.

Problem 1. Find a state feedback control u satisfying $|u| \leq 1$ such that the closed-loop system is globally asymptotically stable and locally exponentially stable at the origin.

We give some explanations on the problem.

Remark 1. Similar to the analysis in Zhou et al. (2018), the linear terms $Ax_i(t-h_i)$ (defined by $\mathcal{L}_i(x_t)), i \in \mathbf{I}[2, p+1]$, in (1) can be replaced by the general ones $\mathcal{L}_i(x_t) = \Sigma_{j=i}^{p+1} \Sigma_{k=1}^{m_{ij}} A_{ijk} x_j(t-h_{ijk}), \; i \in \mathbf{I}[2, p+1]$, where $m_{ij} \geq 1$ are integers, h_{ijk} are known non-negative numbers, and $A_{ijk} \in \mathbf{R}^{2\times 2}$ are some known matrices such that the linearized system of (1) is controllable.

Remark 2. The upper bounds “1” in Assumption 1 and Problem 1 can be replaced by any given positive constant ρ. For example, if we study Problem 1 with $|u| \leq \rho$ for system (1) satisfying Assumption 1, then by the change of variable $v = u/\rho$, the system still satisfies Assumption 1 where the scalars ϕ_i are updated accordingly.

For system (1), when $h_i = 0, i \in \mathbf{I}[2,p]$, and $r = 0$, Problem 1 has been investigated in Ye (2014) based on a canonical form introduced by Teel in Teel (1992). However, since the control is subject to time delay, the decoupling property is no longer valid in the recursive design, which makes the analysis in Ye (2014) rather involved. In this paper, we will construct a novel canonical form containing not only time delay in the input but also time delays in its state, which can keep the decoupling property in the recursive design. With the aid of the special canonical form, a type of nonlinear control laws will be proposed to solve Problem 1. The proposed controllers contain some free parameters that can be designed to improve the control performance. Moreover, such a special canonical form permits us to handle the feedforward system containing delayed states, which was not considered in Ye (2014).

3. A NOVEL STATE SPACE TRANSFORMATION

In this section, we will present a special state space description of system (1), which is crucial in establishing the main results. To this end, we define

$$\tau \geq \sum_{i=2}^{p+1} h_i \geq 0, \tag{4}$$

and

$$A_\tau \triangleq bb^{\mathrm{T}} \mathrm{e}^{A_\omega \tau} = \begin{bmatrix} 0 & 0 \\ -\sin(\omega\tau) & \cos(\omega\tau) \end{bmatrix}. \tag{5}$$

Lemma 1. Let λ be a given positive constant and consider the following linear time-delay system:

$$\begin{cases} \dot{y}_1(t) = A_\omega y_1(t) + \sum_{i=2}^{p} \lambda A_\tau y_i(t-\tau) + bu(t-\tau), \\ \quad\vdots \\ \dot{y}_{p-1}(t) = A_\omega y_{p-1}(t) + \sum_{i=p}^{p} \lambda A_\tau y_i(t-\tau) + bu(t-\tau), \\ \dot{y}_p(t) = A_\omega y_p(t) + bu(t-\tau), \end{cases} \tag{6}$$

where $y=[y_1^{\mathrm{T}},y_2^{\mathrm{T}},\ldots,y_p^{\mathrm{T}}]^{\mathrm{T}}\in\mathbf{R}^{2p}$ with $y_i=[y_{i1},y_{i2}]^{\mathrm{T}}\in\mathbf{R}^2, i\in\mathbf{I}[1,p]$. Then there exists an invertible upper block-triangular transformation $y(t)=\mathcal{T}(x_{[t-\gamma_1,t]})$ (its associated inverse transformation is denoted by $x(t)=\mathcal{G}(y_{[t-\gamma_2,t+\gamma_2]})$), in which $\gamma_1=\max\{|\tau_i|,|\tau_{ijk}|\}\geq 0$, $\gamma_2=\max\{|\kappa_i|,|\kappa_{ijk}|\}\geq 0$, such that system (1) with $f_i(\cdot)=0, i\in\mathbf{I}[1,p]$, is transformed into (6) for $t\geq\gamma_1$. Here

$$\begin{aligned}\mathcal{T}&=\begin{bmatrix}\mathcal{T}_{11} & \mathcal{T}_{12} & \cdots & \mathcal{T}_{1p}\\ & \ddots & \ddots & \vdots\\ & & \mathcal{T}_{p-1,p-1} & \mathcal{T}_{p-1,p}\\ & & & \mathcal{T}_{pp}\end{bmatrix},\\ \mathcal{G}&=\begin{bmatrix}\mathcal{G}_{11} & \mathcal{G}_{12} & \cdots & \mathcal{G}_{1p}\\ & \ddots & \ddots & \vdots\\ & & \mathcal{G}_{p-1,p-1} & \mathcal{G}_{p-1,p}\\ & & & \mathcal{G}_{pp}\end{bmatrix},\end{aligned}\tag{7}$$

where $\mathcal{T}_{ij},\mathcal{G}_{ij}, j\in\mathbf{I}[i,n], i\in\mathbf{I}[1,n]$, are linear operators defined by

$$\begin{cases}\mathcal{T}_{ii}\left((x_i)_{[t-\gamma_1,t]}\right)=\Phi_{ii}x_i(t+\tau_i),\\ \mathcal{T}_{ij}\left((x_j)_{[t-\gamma_1,t]}\right)=\displaystyle\sum_{k=1}^{q_{1ij}}\Phi_{ijk}x_j(t+\tau_{ijk}),\end{cases}\tag{8}$$

and

$$\begin{cases}\mathcal{G}_{ii}\left((y_i)_{[t-\gamma_2,t+\gamma_2]}\right)=\Psi_{ii}y_i(t+\kappa_i),\\ \mathcal{G}_{ij}\left((y_j)_{[t-\gamma_2,t+\gamma_2]}\right)=\displaystyle\sum_{k=1}^{q_{2ij}}\Psi_{ijk}y_j(t+\kappa_{ijk}),\end{cases}\tag{9}$$

in which $q_{1ij}\geq 1, q_{2ij}\geq 1$ are some integers, $\Phi_{ii}=\Phi_{ii}(\lambda,\omega,\tau),\Psi_{ii}=\Psi_{ii}(\lambda,\omega,\tau)$ are some invertible $\mathbf{R}^{2\times 2}$ matrices with elements being polynomial functions of $\{\lambda,1/\lambda,\cos(\omega\tau),\sin(\omega\tau)\}$, $\Phi_{ijk}=\Phi_{ijk}(\lambda,\omega,\tau)\neq 0_{2\times 2},\Psi_{ijk}=\Psi_{ijk}(\lambda,\omega,\tau)\neq 0_{2\times 2}$ are some $\mathbf{R}^{2\times 2}$ matrices with elements being polynomial functions of $\{\lambda,1/\lambda,1/\omega,\cos(\omega\tau),\sin(\omega\tau)\}$, $\tau_i\leq 0,\tau_{ijk}\leq 0,\kappa_i\geq 0,\kappa_{ijk}$ are polynomial functions of $\{h_i,\tau\}$ satisfying

$$\begin{cases}\tau_{ijk}\leq\tau_j,\ k\in\mathbf{I}[1,q_{1ij}], i\in\mathbf{I}[1,j-1], j\in\mathbf{I}[2,p],\\ \kappa_{ijk}\leq\kappa_i,\ k\in\mathbf{I}[1,q_{2ij}], j\in\mathbf{I}[i+1,p], i\in\mathbf{I}[1,p-1].\end{cases}\tag{10}$$

The case $p=2$ will be illustrated by an example in Section 5 (see (19)--(21)). By using Lemma 1, a special description of system (1) is given in the following corollary.

Corollary 1. By the transformation $y(t)=\mathcal{T}(x_{[t-\gamma_1,t]})$ in Lemma 1, system (1) is transformed into

$$\begin{cases}\dot{y}_1(t)=A_\omega y_1(t)+g_1(t-\tau)+bu(t-\tau)+l_1,\\ \dot{y}_2(t)=A_\omega y_2(t)+g_2(t-\tau)+bu(t-\tau)+l_2,\\ \quad\vdots\\ \dot{y}_{p-1}(t)=A_\omega y_{p-1}(t)+g_{p-1}(t-\tau)+bu(t-\tau)+l_{p-1},\\ \dot{y}_p(t)=A_\omega y_p(t)+bu(t-\tau)+l_p,\end{cases}\tag{11}$$

for $t\geq\gamma_1$, where $g_j(t-\tau)=\Sigma_{i=j+1}^{p}\lambda A_\tau y_i(t-\tau), j\in\mathbf{I}[1,p-1]$, $l_i=l_i((Y_{i+1})_{[t-\mu,t]})$ with $l_i(\cdot)=[l_{i1}(\cdot),l_{i2}(\cdot)]^{\mathrm{T}}\in\mathbf{R}^2, i\in\mathbf{I}[1,p]$, satisfies, for some positive constants $d=d(\lambda,\omega,\tau)\leq 1$ and $\delta_i=\delta_i(\lambda,\omega,\tau), i\in\mathbf{I}[1,p]$,

$$\left|l_i\left((Y_{i+1})_{[t-\mu,t]}\right)\right|\leq\delta_i\left|Y_{i+1}\right|^2_{[t-\mu,t]},\tag{12}$$

whenever $|Y_{i+1}|_{[t-\mu,t]}\leq d\leq 1$, where $Y_i=(y_i^{\mathrm{T}},y_{i+1}^{\mathrm{T}},\ldots,y_p^{\mathrm{T}},y_{p+1})^{\mathrm{T}}, i\in\mathbf{I}[1,p+1]$, with $y_i=[y_{i1},y_{i2}]^{\mathrm{T}}\in\mathbf{R}^2, y_{p+1}=u\in\mathbf{R}$, and $\mu=\mu(r,h_i)\geq 0$ is a (sufficiently large) constant.

When $0\leq t<\gamma_1$, we set $x(\theta)=0,\theta\in[-\gamma_1,-\gamma_3)$ in (8), where $\gamma_3\triangleq\max_{i\in\mathbf{I}[2,n]}\{h_i\}$. From (8) and (9) we clearly see that: (i) if the controller $u(t)=u(y(t))$ globally stabilizes the y-system (11), it also globally stabilizes the x-system (1); (ii) the controller $u(t)=u(y(t))$ is implementable since $y(t)$ involves only the current and delayed information of the state $x(t)$ in view of $\tau_i\leq 0$ and $\tau_{ijk}\leq 0$. Therefore, it remains to design the stabilizing controller $u(t)=u(y(t))$ for the y-system (11).

Remark 3. For utilizing the special form of the transformation (8)--(9), the eigenvalues of the system matrix of system (1) (also (6)) are all restricted to be $\pm\omega\mathrm{i},\omega\neq 0$. However, if the eigenvalues are different, for instance, $\pm\omega_j\mathrm{i}, j\in\mathbf{I}[1,k]$ with $k\geq 2$, and $\omega_i\neq\omega_j$ if $i\neq j$, the corresponding transformation can not be expressed as (8) and (9) in the time domain, which will cause some difficulties in the analysis of (1).

4. THE GLOBALLY STABILIZING CONTROLLER

With the aid of the above state space description (11), we are ready to give the main results in this paper. For future use, we define

$$A_p=\begin{bmatrix}1 & & & \\ 1 & 1 & & \\ \vdots & \vdots & \ddots & \\ 1 & 1 & 1 & 1\end{bmatrix}_{p\times p},\ c_p=\left|A_p\right|^2,\tag{13}$$

and

$$\begin{cases}A_{\mathrm{c}_1}=A_\omega-\lambda bb^{\mathrm{T}},\\ A_{\mathrm{c}_2}=(I_p\otimes A_\omega)-\lambda\left(A_p\otimes bb^{\mathrm{T}}\right),\end{cases}\tag{14}$$

where λ is a given positive constant. From (14) we can get the characteristic polynomial of $A_{\mathrm{c}_i}, i=1,2$, are $\alpha(s)=s^2+\lambda s+\omega^2$. Since $\lambda>0$ and $\omega^2>0$, the Lyapunov matrix equations $A_{\mathrm{c}_i}^{\mathrm{T}}P_{\mathrm{c}_i}+P_{\mathrm{c}_i}A_{\mathrm{c}_i}=-2I,\ i=1,2$, have unique positive definite solutions $P_{\mathrm{c}_i}, i=1,2$. Define

$$p_{\mathrm{c}_i}^-=\lambda_{\min}\left(P_{\mathrm{c}_i}\right),\ p_{\mathrm{c}_i}^+=\lambda_{\max}\left(P_{\mathrm{c}_i}\right),\ i=1,2.$$

Theorem 1. Let $\beta\in(0,1)$ and λ be two given positive constants satisfying

$$\begin{cases}\left(p_{\mathrm{c}_1}^-\right)^{1/2}\beta^{p-i}-2\lambda\left(p_{\mathrm{c}_1}^+\right)^{3/2}(\lambda\tau\eta_i+\eta_{i-1})>0,\ i\in\mathbf{I}[1,p],\\ p_{\mathrm{c}_2}^--3\lambda^4\tau^2c_p^2\left(p_{\mathrm{c}_2}^+\right)^3>0,\end{cases}\tag{15}$$

with $\eta_i=\Sigma_{j=1}^{i}\beta^{p-j},\ i\in\mathbf{I}[1,p]$, and $\eta_0=0$. Then there exists a positive constant $\varepsilon^\dagger=\varepsilon^\dagger(\beta,\lambda)\in(0,1)$ such that Problem 1 is solved by the controller $u(t)=-u_p(t)$, in which

$$\begin{cases}u_i(t)=\sigma_{\varepsilon_i}\left(\lambda b^{\mathrm{T}}\mathrm{e}^{A_\omega\tau}y_i(t)\right)+u_{i-1}(t),\ i\in\mathbf{I}[2,p],\\ u_1(t)=\sigma_{\varepsilon_1}\left(\lambda b^{\mathrm{T}}\mathrm{e}^{A_\omega\tau}y_1(t)\right),\end{cases}\tag{16}$$

where $\varepsilon_i, i\in\mathbf{I}[1,p]$, are some constants satisfying

$$\varepsilon_i=\beta^{p-i}\varepsilon,\ \forall\varepsilon\in\left(0,\varepsilon^\dagger\right),\ i\in\mathbf{I}[1,p].\tag{17}$$

Remark 4. For given $\omega\neq 0$ and $\tau\geq 0$, (15) can be guaranteed when the parameters β and λ are sufficiently small. The detailed explanation is presented as follows. On the one hand, it is easy to verify that

$$\begin{cases} p_{c_1}^- = \dfrac{\lambda^2+4\omega^2-\lambda\sqrt{\lambda^2+4\omega^2}}{2\lambda\omega^2} \triangleq \dfrac{1}{\lambda}\alpha_1, \\ p_{c_1}^+ = \dfrac{\lambda^2+4\omega^2+\lambda\sqrt{\lambda^2+4\omega^2}}{2\lambda\omega^2} \triangleq \dfrac{1}{\lambda}\alpha_2, \end{cases} \tag{18}$$

where $\alpha_i = \alpha_1(\lambda, \omega, 1/\omega), i = 1, 2$, from which we know that $\alpha_i \in [d_{i1}, d_{i2}], i = 1, 2$, with d_{ij} being some suitable positive constants independent of λ for any $\lambda \in (0, 1)$. Substituting (18) into the first inequalities of (15) gives, for $i \in \mathbf{I}[1, p]$,

$$\beta^{p-i}\left(\left(\frac{\alpha_1}{\lambda}\right)^{\frac{1}{2}} - 2\alpha_2\left(\frac{\alpha_2}{\lambda}\right)^{\frac{1}{2}}(\lambda\tau\delta_i + \delta_{i-1})\right) > 0,$$

where $\delta_i = \Sigma_{j=1}^{i}\beta^{i-j}$, which can be guaranteed by

$$1 - \frac{2\left(\sqrt{d_{22}}\right)^3}{\sqrt{d_{11}}}(\lambda\tau\delta_i + \delta_{i-1}) > 0,$$

which are obviously true when β and λ are sufficiently small. On the other hand, similar to (18), p_2^- and p_2^+ can be expressed as

$$\begin{cases} p_{c_2}^- = \dfrac{1}{\lambda}\alpha_3 = \dfrac{1}{\lambda}\alpha_3\left(\lambda, \omega, \dfrac{1}{\omega}\right), \\ p_{c_2}^+ = \dfrac{1}{\lambda}\alpha_4 = \dfrac{1}{\lambda}\alpha_4\left(\lambda, \omega, \dfrac{1}{\omega}\right), \end{cases}$$

where $\alpha_i \in [d_{i1}, d_{i2}], i = 3, 4$, with d_{ij} being some suitable positive constants independent of λ for any $\lambda \in (0, 1)$. Then the second inequality in (15) can be rewritten as

$$\frac{1}{\lambda}\alpha_3\left(\lambda, \omega, \frac{1}{\omega}\right) - 3\lambda\tau^2 c_p^2\alpha_4\left(\lambda, \omega, \frac{1}{\omega}\right) > 0,$$

which can be guaranteed by

$$1 - \frac{3\tau^2 c_p^2 d_{42}^2}{d_{31}}\lambda^2 > 0,$$

which is also obviously true when λ is sufficiently small where we have noticed that c_p are some constants independent of λ (see (13)).

5. AN ILLUSTRATIVE EXAMPLE

To illustrate Theorem 1, we consider the following nonlinear system:

$$\begin{cases} \dot{x}_{11}(t) = x_{12}(t) + x_{21}^2(t - r_1), \\ \dot{x}_{12}(t) = -x_{11}(t) + x_{22}(t - h_2) + x_{22}^2(t - r_2), \\ \dot{x}_{21}(t) = x_{22}(t), \\ \dot{x}_{22}(t) = -x_{21}(t) + u(t - h_3). \end{cases} \tag{19}$$

Our design consists of the following two steps.

Step 1: The transformation calculation from x-system to y-system. Based on Lemma 1, the transformation is given by

$$\begin{cases} y_1(t) = \Phi_{11}x_1(t + \tau_1) + \displaystyle\sum_{k=1}^{2}\Phi_{12k}x_2(t + \tau_{12k}), \\ y_2(t) = \Phi_{22}x_2(t + \tau_2) \end{cases} \tag{20}$$

where

$$\Phi_{11} = \begin{bmatrix} \lambda\cos\tau & \lambda\sin\tau \\ -\lambda\sin\tau & \lambda\cos\tau \end{bmatrix}, \Phi_{121} = \begin{bmatrix} -\lambda\sin\tau & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Phi_{122} = \Phi_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and $\tau_1 = -2\tau + h_2 + h_3$, $\tau_{121} = -2\tau + h_3$, $\tau_{122} = \tau_2 = -\tau + h_3$. It is clear see that the above transformation satisfies (8) and the first inequalities of (10) by noticing (4). Moreover, the inverse transformation from y to x is given by

$$\begin{cases} x_1(t) = \Psi_{11}y_1(t + \kappa_1) + \displaystyle\sum_{k=1}^{2}\Psi_{12k}y_2(t + \kappa_{12k}), \\ x_2(t) = \Psi_{22}y_2(t + \kappa_2), \end{cases} \tag{21}$$

where

$$\Psi_{11} = \begin{bmatrix} \dfrac{\cos\tau}{\lambda} & \dfrac{-\sin\tau}{\lambda} \\ \dfrac{\sin\tau}{\lambda} & \dfrac{\cos\tau}{\lambda} \end{bmatrix}, \Psi_{121} = \begin{bmatrix} \dfrac{-\cos\tau}{\lambda} & \dfrac{\sin\tau}{\lambda} \\ \dfrac{-\sin\tau}{\lambda} & \dfrac{-\cos\tau}{\lambda} \end{bmatrix},$$

$$\Psi_{122} = \begin{bmatrix} \dfrac{\sin(2\tau)}{2} & 0 \\ \sin^2\tau & 0 \end{bmatrix}, \Psi_{33} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and $\kappa_1 = \kappa_{121} = 2\tau - (h_2 + h_3)$, $\kappa_{122} = \tau - (h_2 + h_3)$ $\kappa_2 = \tau - h_3$. It is also clear see that the above inverse transformation satisfies (9) and the second inequalities of (10) by noticing (4).

Step 2: Design of $u(t)$ for system (19). Based on the above transformation, the controller proposed in Theorem 1 takes the form

$$u(t) = -\sigma_{\varepsilon_2}\left(\lambda b^{\mathrm{T}}\mathrm{e}^{A_\omega\tau}y_2(t)\right) - \sigma_{\varepsilon_1}\left(\lambda b^{\mathrm{T}}\mathrm{e}^{A_\omega\tau}y_1(t)\right), \tag{22}$$

where $y_i(t), i = 1, 2$, are given by (20), A_ω, b are given by (2) with $\omega = 1$, and $\varepsilon_i, i = 1, 2$, satisfy

$$\varepsilon_i = \beta^{2-i}\varepsilon, \ i = 1, 2. \tag{23}$$

Now we perform simulations for the closed-loop system consisting (19) and (22). Let $h_2 = h_3 = 0.2, \tau = 0.4$, $\varepsilon = 0.6$ and $\beta = 0.15$, $\lambda = 0.1$ (which satisfy (15)). For the simulation purpose, we let the "unknown" time delays be chosen as $r_i = 0.5, i = 1, 2$. For a given initial condition $x(0) = [-1, -1, 1, 1]^{\mathrm{T}}, x(\theta) = [0, 0, 0, 0]^{\mathrm{T}}, \forall\theta \in [-0.6, 0)$, $u(\theta) = 0, \theta \in [-0.2, 0]$, the 2-norm of the state vectors of the closed-loop system consisting (19) and (22) is recorded in Fig. 1. It follows that the states converge to the origin, which indicates asymptotic stability of the closed-loop system.

At last, we show that the stability conditions (15) in Theorems 1 may be conservative. Let $\beta = 0.4$, $\lambda = 0.15$ which do not satisfy (15) and the other values be chosen as above. the 2-norm of the state vectors of the closed-loop system consisting (19) and (22) is recorded in Fig. 2. It follows that the states still converge to the origin.

6. CONCLUSION

This paper has investigated the global stabilization of a family of feedforward nonlinear time-delay systems with linearized identical oscillators with bounded feedback. Based on a special canonical form which contains not only time delay in the input but also time delays in its state, a type of nonlinear control laws was proposed to achieve global stabilization. The proposed nonlinear controllers contain some free parameters that can be designed to improve the control performance. A numerical example was given to show the effectiveness of the proposed methods.

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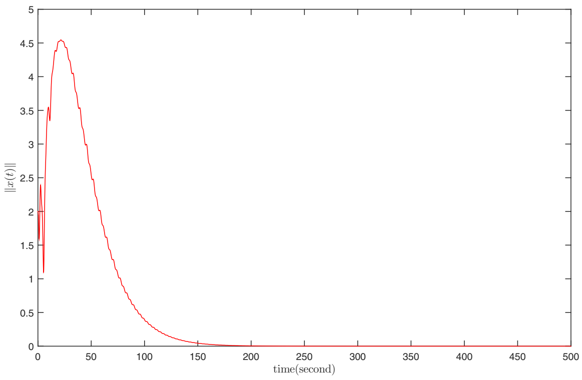

Fig. 1. States in 2-norm of closed-loop system consisting (19) and (22) with $\beta = 0.15$, $\lambda = 0.1$.

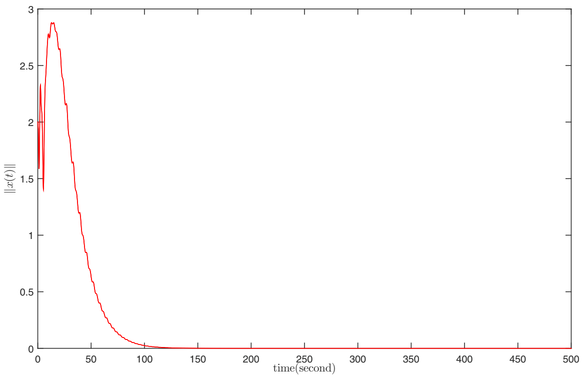

Fig. 2. States in 2-norm of closed-loop system consisting (19) and (22) with $\beta = 0.4, \lambda = 0.15$.

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