Positivity-based approach to stability of delay systems: New trends

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Positivity-based methods are widely used in the stability of systems composed of first-order equations. The positivity of the diagonal terms in all equations of the system was assumed that excluded the possibility of extending these approaches to systems composed of second-order equations. In the following system

$$x^{''}(t)+\sum_{j=1}^{n}a_{ij}(t)x(t-\tau_{ij}(t))-\sum_{j=1}^{n}b_{ij}(t)x(t-\theta_{ij}(t))=0, \quad t\in[0,+\infty), \tag{1}$$

where $a_{ij}(t), b_{ij}(t), \tau_{ij}(t)$ and $\theta_{ij}(t)$ are measurable essentially bounded functions, there are no terms with first derivatives. It was believed that in this case it is impossible to achieve stability even in the case of scalar equation. We denote $q_* = \text{essinf}_{t\geq 0} q(t)$, $\quad q^* = \text{esssup}_{t\geq 0} q(t)$.

Theorem. *Assume that* $0\leq \tau_{ii}(t)<\theta_{ii}(t)$, $0\leq b_{ii}(t)<a_{ii}(t)$, $4\left\{a_{ii}(t)-b_{ii}(t)\right\}\leq \left[b_{ii}(\theta_{ii}-\tau_{ii})\right]_*^2$, $t\in[0,+\infty)$, $0<\left[b_{ii}(\theta_{ii}-\tau_{ii})\right]^*\theta_{ii}^*\leq \frac{1}{e}$. *If there exist a vector* $z=\text{col}\left\{z_1,...,z_n\right\}$ *with positive components* $z_i>0$ *for* $i=1,...,n$ *and a number* $\varepsilon>0$ *such that*

$$(a_{ii}(t)-b_{ii}(t))z_i-\sum_{j=1,j\neq i}^{n}\left|a_{ij}(t)\right|z_j-\sum_{j=1,j\neq i}^{n}\left|b_{ij}(t)\right|z_j>\varepsilon, \quad t\in[0,+\infty), \tag{2}$$

then system (1) is exponentially stable. If in addition $a_{ij}(t)\leq 0$, $b_{ij}(t)\leq 0$ *for* $j\neq i$, $i,j=1,...,n$, *then condition (2) is necessary a sufficient for the exponential stability of system (1).*