Input Delay Effect on Cluster Consensus in Continuous-Time Networks ⋆

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Abstract: In this paper, we investigate the effect of input time-delay on the cluster consensus of multi-agent systems operating in continuous-time. Contrary to the studies on cluster/group consensus in the literature, the clusters are not pre-determined. We investigate the stability of a multi-agent network with fixed input time-delay. The analysis yields the upper bound of time-delay that does not affect the number of clusters and convergence properties of the agents. Theoretical results are illustrated via several simulations.

Keywords: Time-delay, linear systems, continuous-time systems, multi-agent systems, directed graphs, cluster consensus

1. INTRODUCTION

Recently, the distributed consensus problem, one of the most studied in multi-agent systems, has been the center of interest due to a wide range of applications in different disciplines including biology, physics, formation of mobile robots, control engineering, wireless sensor networks (Ren and Beard (2005), Olfati-Saber et al. (2007), Jadbabaie et al. (2003), Akar and Shorten (2008)).

In the literature, there is a vast amount of studies on complete consensus in which all agents of a multi-agent system converge to a single final value. However, it is also possible that agents in a multi-agent system may go to different consistent states which are called *clusters*. Every cluster attains a consensus state which differs from those reached by other clusters. This phenomenon is referred to as *cluster consensus*.

In this paper, the focus is to study the effect of input delay that may lead to instability in a multi-agent system. Some related studies which are on either complete consensus or cluster consensus are briefly reviewed below.

Olfati-Saber and Murray (2004) study consensus problems in networks of agents with or without time-delays. Multi-agent systems they investigate can be under fixed or switching topologies, directed or undirected information exchange. They derive the maximum time-delay that can be tolerated by a multi-agent system. The maximum time-delay is inversely proportional to the largest eigenvalue of the Laplacian system matrix. However, the result on time-delay in the study is only valid for single consensus multi-agent systems evolving over undirected graphs.

Lin and Jia (2008) investigate the average-consensus problem of multi-agent systems that can be under switching topologies and time-delays in continuous-time networks. Time delay can be constant or time-varying. They propose a Lyapunov-Krasovskii function to guarantee average-consensus of the agents under arbitrary switching topology. Sufficient conditions are stated in terms of linear matrix inequalities (LMIs) for directed networks.

Chen et al. (2017) study the delay effect on group consensus of second-order multi-agent systems. They consider a multi-agent network as an interconnection of two sub-groups and construct a consensus protocol for each sub-group to reach two different final values. They not only obtain consensus conditions without time-delays, but also delay-dependent conditions are stated by frequency-domain analysis.

Hu and Hong (2007) investigate leader-following coordination problem of multi-agent systems with coupling time-delays. In the multi-agent systems they discuss, agents have second-order dynamics, connections of agents are directed, and coupling-time delay is time varying. Lyapunov-Razumikhin functions are used to deal with convergence and stability problems of multi-agent systems with coupling-time delays in continuous-time networks.

In this paper, we investigate the effect of input delay on a multi-agent system that has cluster consensus property. The main aim of the paper is to determine the interval of fixed input time-delay that has no effect on the number of clusters and does not change the elements of the groups. Moreover, we do not artificially divide the system into clusters. We extend the study (Develer and Akar (2018)) that is discussed for continuous-time networks without time-delay.

The rest of the paper is organized as follows. In Section 2, we review some concepts from graph theory and formulate the distributed cluster consensus problem. In Section 3, the effect of input delay on multi-agent systems which converge to clusters is investigated. Simulation studies are given in Section 4. Finally, there are some concluding remarks in Section 5.

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2. MATHEMATICAL PRELIMINARIES

Let $G = (V, \mathcal{E}, A)$ be a weighted directed graph (digraph) that represents the interaction topology of a multi-agent system with n nodes. The digraph consists of the set of nodes $V = \{v_1, v_2, \ldots, v_n\}$, set of edges $\mathcal{E} \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ with nonnegative elements. The node indices belong to a finite index set $I = \{1, 2, \ldots, n\}$. A directed edge is denoted by $e_{ij} = (v_j, v_i)$ and exists if node v_i receives information from node v_j. The adjacency element, a_{ij}, is the weight for the edge, e_{ij}, and positive, i.e., $e_{ij} \in \mathcal{E} \Leftrightarrow a_{ij} > 0$ $\forall i, j \in I$. In this paper, we assume that all nodes have self loops, i.e., $a_{ii} > 0$ for all $i \in I$. The set of neighbors of node v_i is defined by $N_i = \{v_j | e_{ij} \in \mathcal{E}\}$. A digraph is said to be *strongly connected*, if every node can reach every other node. If there exists at least one node that can reach other nodes in the digraph, the digraph has a *spanning tree.*

The *degree matrix* is an $n \times n$ diagonal matrix defined as $\Delta = [\Delta_{ij}] = diag\{\Delta_{ii}\}$. The in-degree of node v_i, denoted by $deg_{in}(v_i)$, is the number of its inward edges and the out-degree of node v_i, denoted by $deg_{out}(v_i)$, is the number of its outward edges. $deg_{in}(v_i)$ and $deg_{out}(v_i)$ are defined as follows:

$$\deg_{\text{in}}(v_i) = \sum_{j=1}^{n} a_{ij}, \quad \deg_{\text{out}}(v_i) = \sum_{j=1}^{n} a_{ji}. \tag{1}$$

The degree matrix for a digraph is defined as

$$\Delta_{ij} := \begin{cases} deg_{in}(v_i), & i = j, \\ 0, & i \neq j. \end{cases} \tag{2}$$

The Laplacian of G is defined by

$$L = \Delta - A. \tag{3}$$

2.1 Distributed Consensus

Consider a multi-agent system consisting of n agents with linear dynamics in continuous-time. The dynamics of node v_i are described by

$$\dot{x}_i(t) = u_i(t), \quad i \in I \tag{4}$$

where $x_i(t)$ is the state of node v_i at time t, and $u_i(t)$ is the system input described as follows:

$$u_i(t) = \sum_{j \in N_i} a_{ij}(x_j(t) - x_i(t)), \quad i \in I. \tag{5}$$

Assumption 1. $a_{ij} > 0$ if $e_{ij} \in \mathcal{E}$, and $a_{ij} = 0$ if $e_{ij} \notin \mathcal{E}$ for $i = 1, 2, \ldots, n$.

Assumption 1 implies that the information coming from a neighbor should have positive weighting.

The dynamics of the multi-agent system (4) with the system input (5) is equivalent to

$$\dot{x}(t) = -Lx(t) \tag{6}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the state vector of the digraph G at time t, and $L = \Delta - A = [l_{ij}] \in R^{n\times n}$ is defined by

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} a_{ik}, & \text{if } i = j \\ -a_{ij}, & \text{otherwise.} \end{cases} \tag{7}$$

If Assumption 1 holds, $l_{ii} \geq 0$ and $l_{ij} \leq 0$.

Remark 1. A crucial property of the Laplacian matrix, L, is that each row of L adds up to zero. Then, $\pi = [1, 1, \ldots, 1]^T \in R^n$ is an eigenvector of L associated with the eigenvalue $\lambda = 0$, i.e., Lπ=0.

2.2 Cluster Consensus States

In this part, we introduce the definition of the cluster consensus problem that will be studied in this paper and review some definitions that will be useful to determine the number of clusters.

Definition 2. (Cluster Consensus) The network in (4) is said to go to K disjoint clusters, $C = \{C_1, C_2, \ldots, C_K\}$, if it satisfies the following properties for any initial condition $[x_1(0), x_2(0), \ldots, x_n(0)]^T$ and for all weighted adjacency elements that obey Assumption 1:

- $\bigcup_{p=1}^{K} C_p = V$,
- $C_p \cap Cq = \emptyset$, for $p \neq q$, and $p, q = 1, 2, \ldots, K$,
- $\lim_{t\to\infty} x_i(t) = c_p$, $\forall v_i \in C_p$, $i = 1, 2, \ldots, n$, and $c_p \neq c_q$ for $p \neq q$, $p, q = 1, 2, \ldots, K$.

To investigate the cluster consensus problem for directed networks, the concepts of primary and secondary layer subgraphs are used.

Definition 3. (Erkan et al. (2018)) (Primary layer subgraphs) Let *G=(V,$\mathcal{E}$)* be the graph. There exist l_1 ($l_1 \geq 1$) subsets in the vertex set V such that each subset $V_{1,i}$, $i = 1, 2, ..., l_1$, is the largest possible subset that has a spanning tree for its subgraph $G_{1,i}$, and for all $v_a \in V_{1,i}$ and $v_b \notin V_{1,i}$, we have $(v_a, v_b \notin \mathcal{E})$. We say $G_{1,i}$, $i = 1, 2, .., l_1$ are the primary layer subgraphs of G where the number of primary layer subgraphs is denoted by l_1.

Remark 4. For a network, the primary layer subgraphs of the graph are denoted as $G_{1,1}, G_{1,2}, \ldots, G_{1,l_1}$.

Definition 5. (Erkan et al. (2018)) (Secondary layer subgraph) Let $\bar{V}$ be the set which consists of the nodes that are not in the primary layer subgraphs, i.e., $\bar{V} = V \setminus \bigcup_{i=1}^{l_1} V_{1,i}$. Then there exist l_2 subsets in $\bar{V}$ such that each subset $V_{2,i}$, $i = 1, 2, ..., l_2$, has a spanning tree for its subgraph $G_{2,i}$ and for the root of this spanning tree, $v_a \in V_{2,i}$, there exist at least two nodes in two different subgraphs (either primary or secondary layer) v_b and v_c such that $(v_b, v_a) \in \mathcal{E}$ and $(v_c, v_a) \in \mathcal{E}$. For all $v_d \in V_{2,i} \setminus v_a$ and $v_e \in V \setminus V_{2,i}$, we have $(v_e, v_d) \notin \mathcal{E}$. We define the subset $V_{2,i}$, $i = 1, 2, ..., l_2$ as the secondary layer subgraphs of G.

Remark 6. For a network, the secondary layer subgraphs of the graph are denoted as $G_{2,1}, G_{2,2}, \ldots, G_{2,l_2}$.

The primary and secondary layer subgraphs of a given network can be determined by applying the algorithm of Erkan et al. (2018). Once these subgraphs are obtained, the following lemma gives the number of clusters in a continuous-time multi-agent network.

Lemma 7. (Develer and Akar (2018)) The number of clusters for a continuous-time multi-agent network with digraph $G = (V, \mathcal{E})$ can be computed as

$$K = l_1 + l_2 \tag{8}$$

where l_1 and l_2 are the number of primary and secondary layer subgraphs, respectively.

3. CLUSTER CONSENSUS WITH TIME-DELAYS

The objective of this paper is to investigate the effect of fixed input time-delay on directed network topology such that the network goes to K ≥ 2 clusters. If there exists a fixed input time-delay on a system input, (4) becomes

$$\dot{x}_i(t) = u_i(t - \tau_d) \tag{9}$$

where

$$u_i(t-\tau_d) = \sum_{j \in N_i} a_{ij}(x_j(t-\tau_d) - x_i(t-\tau_d)), \quad i \in I, \tag{10}$$

and τ_d is the fixed input time-delay.

By taking the Laplace transform of the above equation, we obtain

$$\begin{aligned} sX_i(s) - x_i(0) &= U_i(s)e^{-\tau_d s} \\ &= \sum_{j \in N_i} a_{ij}(X_j(s) - X_i(s))e^{-\tau_d s} \end{aligned} \tag{11}$$

where $X_i(s)$ and $U_i(s)$ are the Laplace transforms of $x_i(t)$ and $u_i(t)$, respectively.

Equation (11) can also be rewritten in matrix form as

$$X(s) = (sI + e^{-\tau_d s}L)^{-1}x(0). \tag{12}$$

Now, the analysis of cluster consensus for a multi-agent system with time-delay becomes a stability problem of the following MIMO (multiple-input multiple-output) transfer function:

$$H(s) = (sI + e^{-\tau_d s}L)^{-1}. \tag{13}$$

Theorem 8. A multi-agent network with fixed input time-delay $\tau_d > 0$ is stable if and only if the following condition is satisfied:

- $\tau_d \in (0, \tau^*)$ with $\tau^* = \min_k \Big[\frac{1}{|\lambda_k|} \arctan\Big(\Big|\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}}\Big|\Big)\Big]$ where λ_k is a nonzero eigenvalue of L.

Furthermore, the network results in $l_1 + l_2$ clusters where l_1 and l_2 are the number of primary and secondary layer subgraphs, respectively.

Proof. There are two parts of the proof which are stability of a multi-agent system with fixed input time-delay and the effect of the delay on the number of clusters.

i) Stability of a multi-agent system with fixed input time-delay:

From Olfati-Saber and Murray (2004), the condition for stability of the MIMO transfer function, $H(s)$, is

$$s = 0 \text{ or } s + \lambda_k e^{-\tau_d s} = 0 \text{ for } s \neq 0. \tag{14}$$

Unlike the analysis by Olfati-Saber and Murray (2004), eigenvalues of L can be complex since the graph of the system is directed, i.e. $\lambda_k = Re\{\lambda_k\} + jIm\{\lambda_k\}$. In order to determine the upper bound of time-delay τ_d, we need to get the smallest value of τ_d that is larger than zero such that (14) has a zero on the imaginary axis. Then, (14) can be rewritten as

$$jw + (Re\{\lambda_k\} + jIm\{\lambda_k\})e^{-j\tau_d w} = 0. \tag{15}$$

Expanding (15) yields

$$jw + (Re\{\lambda_k\} + jIm\{\lambda_k\})(cos(\tau_d w) - jsin(\tau_d w)) = 0$$

$$\begin{aligned} jw + Re\{\lambda_k\}cos(\tau_d w) - jRe\{\lambda_k\}sin(\tau_d w) + \\ jIm\{\lambda_k\}cos(\tau_d w) + Im\{\lambda_k\}sin(\tau_d w) = 0. \end{aligned} \tag{16}$$

Real and imaginary parts of (16) must be zero individually:

$$j(w - Re\{\lambda_k\}sin(\tau_d w) + Im\{\lambda_k\}cos(\tau_d w)) = 0. \tag{17}$$

$$Re\{\lambda_k\}cos(\tau_d w) + Im\{\lambda_k\}sin(\tau_d w) = 0. \tag{18}$$

From (18), we have

$$\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}} = -\frac{sin(\tau_d w)}{cos(\tau_d w)} = -tan(\tau_d w). \tag{19}$$

Substituting (19) into (17), we obtain

$$\begin{aligned} &w = Re\{\lambda_k\}sin(\tau_d w) - Im\{\lambda_k\}cos(\tau_d w) \\ &Re\{\lambda_k\} = -wcos(\tau_d w) \\ &Im\{\lambda_k\} = wsin(\tau_d w) \\ &w^2 = Re\{\lambda_k\}^2 + Im\{\lambda_k\}^2 \\ &w = \pm|\lambda_k|. \end{aligned} \tag{20}$$

After finding w in terms of λ_k, from (19) we get

$$\begin{aligned} \tau_d w &= \arctan\Big(-\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}}\Big) \\ \tau_d &= \frac{1}{w}\arctan\Big(-\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}}\Big) \\ \tau_d &= \frac{1}{\pm|\lambda_k|}\arctan\Big(-\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}}\Big). \end{aligned} \tag{21}$$

We know that non-zero eigenvalues of the Laplacian matrix have positive real-parts and $\tau_d > 0$. So, if $Im\{\lambda_k\} > 0$, $w = -|\lambda_k|$, otherwise $w = |\lambda_k|$. Then, the smallest value of $\tau^* > 0$ satisfies the following equation:

$$\tau^* = \min_k \Big[\frac{1}{|\lambda_k|}\arctan\Big(\Big|\frac{Re\{\lambda_k\}}{Im\{\lambda_k\}}\Big|\Big)\Big]. \tag{22}$$

Moreover, if $\tau_d = \tau^*$ holds, the response of the system becomes oscillatory.

ii) The effect of the time-delay on the number of clusters:

In the previous part, the interval of time-delay that can be tolerated by a multi-agent system is given. We now investigate the number of clusters in the multi-agent system with fixed input time-delay.

Let $I_1 = \{1, 2, \ldots, l_1\}$ and $I_2 = \{1, 2, \ldots, l_2\}$ be the index sets of the primary and secondary layer subgraphs, respectively. $n_{1,i}$, $i \in I_1$ denotes the number of nodes in the i-th primary layer subgraph and $n_{2,j}$, $j \in I_2$ denotes the number of nodes in the j-th secondary layer subgraph. Let $\tilde{n}_1 = \sum_{i=1}^{l_1} n_{1,i}$ and $\tilde{n}_2 = \sum_{j=1}^{l_2} n_{2,j}$ be the total number of nodes in the primary layer subgraphs and the secondary layer subgraphs, respectively. Based on the primary and secondary layer decompositions of the network, the delayed consensus system can be represented as

$$\dot{x}_p(t) = -L_p x_p(t - \tau_d) \tag{23a}$$

$$\dot{x}_s(t) = -L_s x_s(t - \tau_d) - L_{sp} x_p(t - \tau_d) \tag{23b}$$

where $x_p(t) \in R^{\tilde{n}_1 \times 1}$ and $x_s(t) \in R^{\tilde{n}_2 \times 1}$ are the state vectors for the primary and secondary layer subgraphs at time t, respectively; and the system matrices are given as

$$L_p = \begin{bmatrix} L_{1,1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & L_{l_1,l_1} \end{bmatrix}_{\tilde{n_1} \times \tilde{n}_1},$$

$$L_{sp} = \begin{bmatrix} L_{l_1+1,1} & \dots & L_{l_1+1,l_1} \\ \vdots & \ddots & \vdots \\ L_{l_1+l_2,1} & \dots & L_{l_1+l_2,l_1} \end{bmatrix}_{\tilde{n_2} \times \tilde{n}_1},$$

$$L_s = \begin{bmatrix} L_{l_1+1,l_1+1} & \dots & L_{l_1+1,l_1+l_2} \\ \vdots & \ddots & \vdots \\ L_{l_1+l_2,l_1+1} & \dots & L_{l_1+l_2,l_1+l_2} \end{bmatrix}_{\tilde{n}_2 \times \tilde{n}_2}.$$

We investigate separately the primary and secondary layer dynamics.

L_p contains l_1 Laplacian matrices corresponding to l_1 primary layer subgraphs. Each primary layer subgraph dynamics can be expressed as

$$\dot{x}_{p,i}(t) = -L_{i,i} x_{p,i}(t - \tau_d) \quad i = 1, 2, \dots, l_1$$

where $x_{p,i}$ is the state vector for the i-th primary layer subgraph. Each primary layer subgraphs with time-delay reaches consensus which results in l_1 clusters in l_1 primary layer subgraphs since each primary layer subgraph is independent of each other.

The equilibrium point of the secondary layer dynamics in (23b) can be calculated as

$$\bar{x}_s = -L_s^{-1} L_{sp} \bar{x}_p \tag{24}$$

where $\bar{x}_p$ and $\bar{x}_s$ are the equilibrium points of the delayed primary and secondary layer subgraphs, respectively. Develer and Akar (2018) show that L_s is well-defined. Also, it can be demonstrated that each row of $L_s^{-1} L_{sp}$ adds up to -1.

The number of clusters in the delayed secondary layer subgraphs is l_2 since the number of the delayed primary layer subgraphs does not change.

Consequently, a multi-agent system with fixed input time-delay is stable if and only if $\tau_d \in (0, \tau^*)$ and the time-delay does not affect the number of clusters in the system satisfying Lemma 7.

Remark 9. In case all eigenvalues of L are real, i.e., $Im\{\lambda_k\} = 0$ for all k, from (22) we have $\tau^* = \min_k \frac{\pi}{2\lambda_k}$ which agrees with Olfati-Saber and Murray (2004).

4. SIMULATION ANALYSIS

Consider the network with 12 nodes and 19 edges in Fig. 1. The state values for the network without any time-delay are shown in Fig. 2 for randomly selected initial state values, and the following Laplacian matrix:

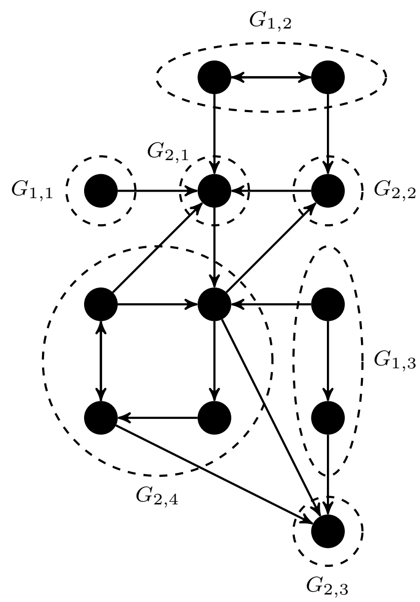

Fig. 1. A directed graph with 12 nodes and 19 edges

$$L = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -13 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -13 & 33 & 0 & 0 & -12 & 0 & 0 & -8 \\
-13 & -2 & 0 & 0 & 0 & 0 & 40 & -10 & 0 & 0 & -5 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 17 & -16 & 0 & 0 & 0 \\
0 & 0 & 0 & -14 & 0 & 0 & -11 & 0 & 42 & 0 & -17 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -10 & 10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 17 & -17 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -17 & -6 & 23
\end{bmatrix} \tag{25}$$

The above Laplacian matrix is randomly formed from the elements of the weighted adjacency matrix, a_{ij}, which satisfy Assumption 1. As shown in Fig. 2, the network converges to 7 clusters, 3 of which are from primary layer subgraphs and the other 4 of which are from secondary layer subgraphs. Clusters are labelled as $G_{1,1}, G_{1,2}, G_{1,3}$ for the primary layer subgraphs and $G_{2,1}, G_{2,2}, G_{2,3}, G_{2,4}$ for the secondary layer subgraphs.

4.1 Case with $\tau_d < \tau^*$

The eigenvalues of the Laplacian matrix used for the network in Fig. 1 are (33, 43.1021 + j7.0639, 43.1021 - j7.0639, 2.4858, 12, 19.9808, 20.1647 + j5.4505, 20.1647 - j5.4505, 13, 0, 0, 0). From Theorem 8, we can obtain the smallest $\tau_d = \tau^*$ that is larger than zero as 0.03224 for $\lambda_k = 43.1021 - j7.0639$, $w = |\lambda_k|$ or $\lambda_k = 43.1021 + j7.0639$, $w = -|\lambda_k|$.

In the case where τ_d is selected as 0.02500, i.e., τ_d is below the upper bound, the state values are shown in Fig. 3. As it can be seen, the system reaches cluster consensus without any change in the clusters.

Fig. 4 shows the state values of the network with input time-delay, τ_d, that is equal to 0.0300. Still, there is no change in any cluster. It implies that if $\tau_d < \tau^*$ holds, the cluster agreement is achieved.

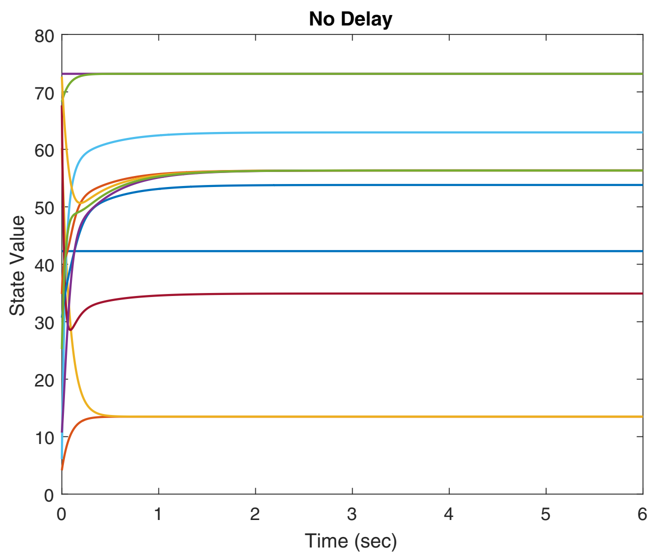

Fig. 2. Simulation results of the network in Fig. 1 without any input time-delay

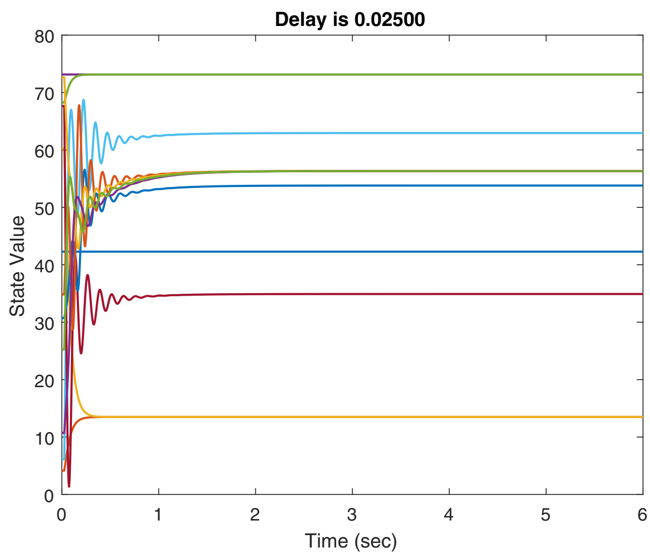

Fig. 3. Simulation results of the network in Fig. 1 with input time-delay $\tau_d = 0.0250$

4.2 Case with $\tau_d = \tau^*$

For the upper bound $\tau_d = \tau^* = 0.03224$, the simulation results are depicted in Fig. 5 which shows the oscillatory behaviour as expected.

4.3 Case with $\tau_d > \tau^*$

The result is illustrated in Fig. 6 for the network with $\tau_d = 0.03500 > \tau^* = 0.03224$. Exceeding the limit, τ^*, brings instability to the network.

4.4 Case with Time-Varying Delay

Now, consider the case where the input time-delay is time varying but does not exceed the limit, τ^*, e.g. $\tau_d(t) = 0.016(1 + sin(2\pi t)) < \tau^*$ where $\tau_d(t)$ is shown in Fig. 7.

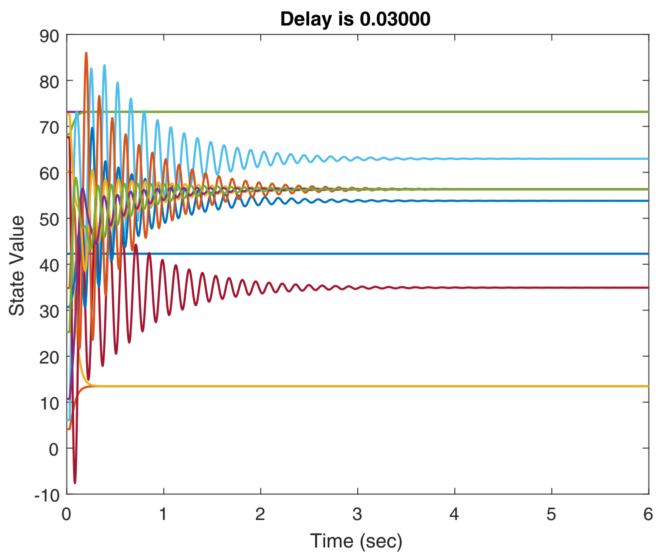

Fig. 4. Simulation results of the network in Fig. 1 with input time-delay $\tau_d = 0.03000$

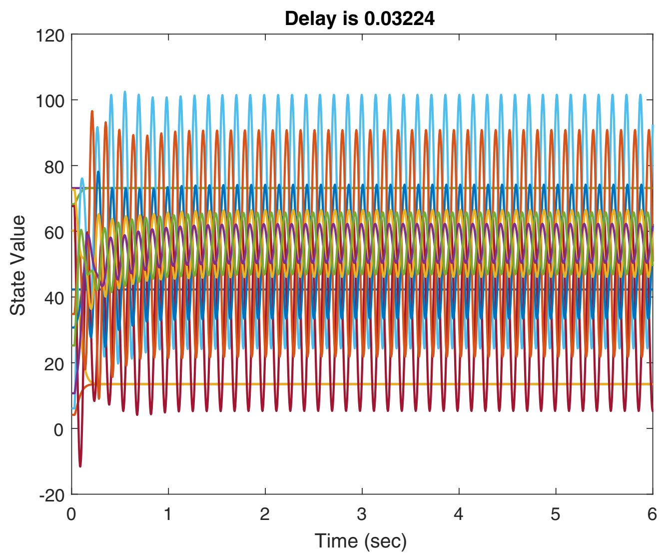

Fig. 5. Simulation results of the network in Fig. 1 with input time-delay $\tau_d = 0.03224$

As shown in Fig. 8, the system exhibits stable behaviour, which is to be theoretically justified.

5. CONCLUSION

In this paper, we have discussed the effect of fixed input time-delay on continuous-time multi-agent networks. We have presented a theorem which gives the interval of time-delay that can be tolerated by a multi-agent system before it becomes unstable.

Our ongoing study is to investigate the effect of communication time-delay which can be caused by information propagation from one node to another in continuous-time networks.

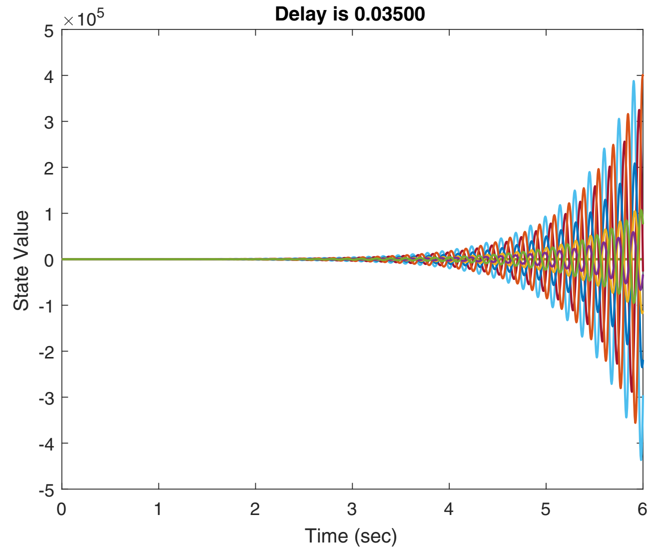

Fig. 6. Simulation results of the network in Fig. 1 with input time-delay $\tau_d = 0.0350$

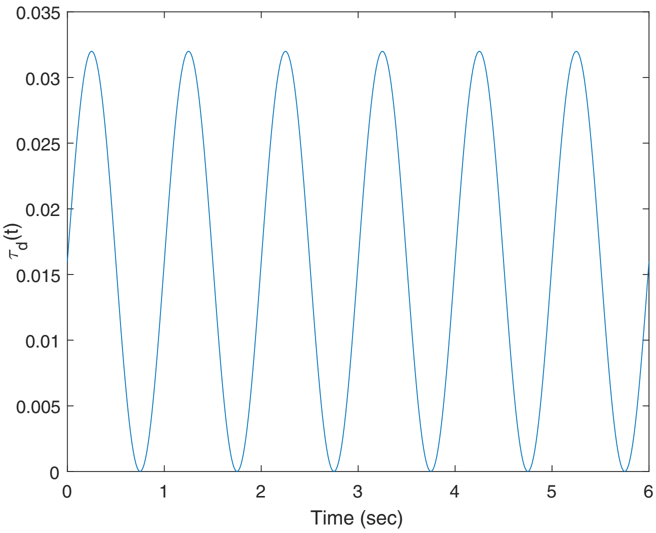

Fig. 7. Input time-delay $\tau_d(t) = 0.016(1 + sin(2\pi t))$

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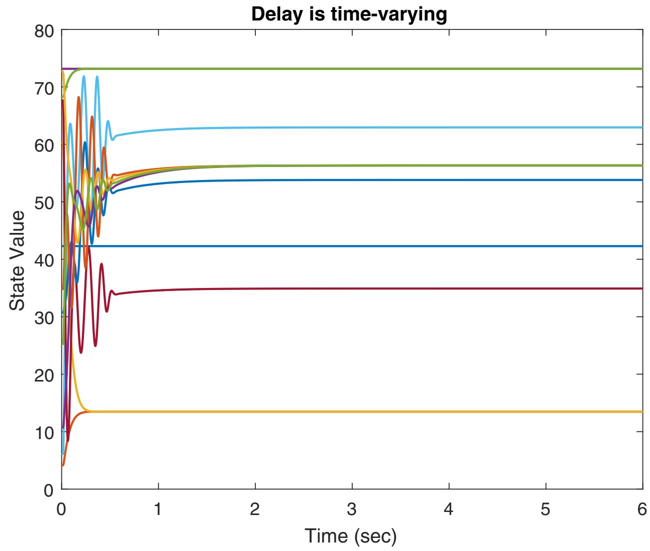

Fig. 8. Simulation results of the network in Fig. 1 with input time-delay $\tau_d(t) = 0.016(1 + sin(2\pi t))$

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