Neutral PID Control Loop Investigated in Terms of Similarity Theory

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Abstract: A study to which extent of parameter setting the PID controller can be designed by means of dominant pole assignment for non-minimum-phase plant with delay is dealt with in the paper. Two options of the control loop are investigated, namely with and without the measurement filtering where the latter leads to a *neutral time delay system* which therefore has to satisfy the *strong stability* condition. The control loop is described in dimensionless quantities obtained by dimensional analysis. The influence of filtering on the loop dynamics is investigated particularly the effect of filter absence at all. This effect is demonstrated by the position of neutral root chain, i.e. how far from the origin lie the roots on a vertical line parallel to the imaginary axis of complex plane. To show boundaries of the dominant three-pole placement possibility in the control loop the trio of poles is placed repeatedly as long as the neutral root chain crosses the imaginary axis and the boundaries are mapped in the space of similarity numbers.

Keywords: neutral time delay systems, PID controller, dominant pole placement, similarity theory, measurement filter.

1. INTRODUCTION

The first order plant with delay is commonly used as an approximation of higher order dynamics. But its PID control loop results in a retarded time delay system only if at least a first order filter is involved in the controller (Åström and Hägglund, 2004; Larsson and Hägglund, 2011). The filter free application of PID to this plant always results in a *neutral character* of the control loop system (Hwang 1993; Kharitonov and Zhabko, 1994). In case of the non-minimum phase process of the second-order with right-hand-part (RHP) zero the PID control loop with delay becomes the system of neutral type (Wang et al. 2017). Typically, neutral time delay systems are observed when controlling communication systems (Olgac and Sipahi, 2004; Olgac, Vyhlídal, and Sipahi, 2008; Castaños et al., 2017). Both retarded and neutral control loop descriptions are presented in Hohenbichler (2009) where the strong stability notion is applied to generate all stabilizing PID controllers.

The dominant pole placement is a powerful concept of PID controller design (Persson and Åström, 1993; Wang et al., 2009). This concept has been successfully applied to the control of time delay systems (Hwang and Fang, 2009; Zítek, Fišer, and Vyhlídal, 2013; Srivastava et al., 2016). The dominant pole placement technique has also been supplemented by a root locus application (Michiels *et al*., 2002; Wang et al., 2009). In He and Fong (2012) an eigenvalue-loci approach is proposed testing a posteriori the rightmost root on its decay rate. Due to the PID controller classification as the three term controller, for more details see Keel and Bhattacharyya (2008), just the meeting of dominant three-pole placement is a natural requirement on the control loop (Hwang 1993; Hwang and Fang, 1995). The dominant three-pole placement for retarded systems is presented in Zítek, Fišer, and Vyhlídal (2013) where the dominance guarantee of the trio of placed poles is based on argument increment principle.

To generalize the control loop description the similarity theory or dimensional analysis is applied (Balaguer 2013). The PID control loop with delay has been designed using the similarity theory reducing the number of process parameters (Zítek, Fišer, and Vyhlídal, 2013). Advanced control loop description based on the dimensional analysis is presented in Zítek, Fišer, and Vyhlídal (2017) where the IAE optimum dominant poles are placed to tune PID controller gains. There are also intuitive approaches to dimensionless description, e.g. Hwang (1993) or non-dimensionalization in Castaños et al. (2017).

When controlling the second-order process with the delay and RHP zero by the PID controller it is well-known the resulting control loop is rendered neutral. The so-called characteristic root problem of general quasi-polynomial functions, both retarded and neutral, was solved already in late forties by Chebotarev and Meiman (1949). In case of neutral one beside the root dominance also the strong stability is to be examined. The strong stability or strong stabilization problem attracted many researchers in the past, for more details see Olgac, Vyhlídal, and Sipahi (2008). To avoid a neutral root chain being close to the imaginary axis and thus preventing these large roots from the influence of the pole dominance the neutral control loop is worth to be investigated. In this point the presented paper extends the results of those papers Zítek, Fišer and Vyhlídal (2017), Fišer, Zítek and Vyhlídal (2017), related to retarded systems.

The paper presents a study to which extent of parameter setting the PID controller can be designed for the non-minimum phase processes of the second-order by means of a dimensionless description. Particularly the effect of the measurement filtering on the stability of the control loop system is investigated.

2. DIMENSIONLESS CONTROL LOOP DESCRIPTION

Consider the second-order process with the delay and an RHP zero (and zero initial conditions) as follows

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + y(t) = K\left[-b_1 \frac{du(t-\tau)}{dt} + u(t-\tau)\right] + C d(t-\tau) \tag{1}$$

where a_1, a_2, b_1 are process model coefficients and τ is the process time delay. K and C are steady-state gains corresponding to the control variable u and disturbance d, respectively. Process (1) is controlled by the PID controller with filtered control error

$$\frac{du(t)}{dt} = r_P \frac{de_f(t)}{dt} + r_I e_f(t) + r_D \frac{d^2 e_f(t)}{dt^2} \tag{2}$$

where r_P, r_I, r_D are proportional, integration and derivative gain, respectively. The control error, e, is filtered as follows

$$T_f \frac{de_f(t)}{dt} + e_f(t) = e(t) \tag{3}$$

under zero initial conditions.

To investigate the impact of measurement filtering on the PID control loop behaviour the following transfer function description is considered

$$G_u(s) = \frac{K\left(-b_1 s + 1\right)e^{-s\tau}}{a_2 s^2 + a_1 s + 1} = \frac{Y(s)}{U(s)}, \tag{4}$$

$$G(s) = \frac{C e^{-s\tau}}{a_2 s^2 + a_1 s + 1} = \frac{Y(s)}{D(s)}, \tag{5}$$

$$R(s) = \frac{r_P s + r_I + r_D s^2}{s} = \frac{U(s)}{E_f(s)}, \tag{6}$$

$$F(s) = \frac{1}{T_f s + 1} = \frac{E_f(s)}{E(s)}. \tag{7}$$

Then using (4) through (7) the disturbance transfer function of the PID control loop is given as $T(s) = G(s) \big/ \left(1 + G_u(s) R(s) F(s)\right)$. Thus this transfer function results

$$T(s) = \frac{C\left(T_f s + 1\right) s e^{-s\tau}}{s\left(a_2 s^2 + a_1 s + 1\right)\left(T_f s + 1\right) + K\left(-b_1 s + 1\right)\left(r_P s + r_I + r_D s^2\right)e^{-s\tau}}. \tag{8}$$

from where the characteristic quasi-polynomial is as follows

$$M(s) = a_2 T_f s^4 + \left(a_2 + a_1 T_f - b_1 K r_D e^{-s\tau}\right)s^3 + \left(a_1 + T_f + K\left(r_D - b_1 r_P\right)e^{-s\tau}\right)s^2 + \left(1 + K\left(r_P - b_1 r_I\right)e^{-s\tau}\right)s + K r_I e^{-s\tau}. \tag{9}$$

To obtain the most general results of the control loop stability investigation by the dimensional analysis is applied to the control loop description in the next subsection.

2.1 Dimensional considerations

A dimensional investigation of PID control loop has been presented in Zítek, Fišer and Vyhlídal (2017). In a similar manner the following nine dimensional quantities

$$\omega\left[\mathrm{s}^{-1}\right], t[\mathrm{s}], Kr_I\left[\mathrm{s}^{-1}\right], Kr_D[\mathrm{s}], T_f[\mathrm{s}], \tau[\mathrm{s}], a_1[\mathrm{s}], b_1[\mathrm{s}], a_2\left[\mathrm{s}^2\right]$$

describing the above control loop are considered in dimensional investigation. The following three similarity numbers may then be introduced for process model (4),

$$\lambda = \sqrt{a_2} / a_1, \; \vartheta = \tau / \sqrt{a_2}, \; \beta = b_1 / \sqrt{a_2}. \tag{10}$$

and similarly the dimensionless parameters

$$\rho_D = Kr_D / \sqrt{a_2}, \; \rho_I = Kr_I \sqrt{a_2}, \; f = T_f / \sqrt{a_2} \tag{11}$$

are introduced for the controller and the filtering. For consistency the dimensionless variables $\bar{t} = t / \sqrt{a_2}$ and $\nu = \omega\sqrt{a_2}$ are introduced for time and frequency, respectively. ν, as dimensionless frequency, is referred to as *frequency angle*. Once the ultimate frequency of the plant (1), ω_K, is assessed the ultimate frequency angle is defined as

$$\nu_K = \omega_K \sqrt{a_2}. \tag{12}$$

The proportional gain $\rho_P = Kr_P$, as dimensionless, is not involved in the dimensional treatment.

With the above introduced quantities the control loop transfer function (8) is rearranged to the form

$$T(\bar{s}) = \frac{C\left(f\bar{s} + 1\right)\bar{s}e^{-\bar{s}\vartheta}}{\bar{s}\left(\bar{s}^2 + \lambda^{-1}\bar{s} + 1\right)\left(f\bar{s} + 1\right) + \left(-\beta\bar{s} + 1\right)\left(\rho_P\bar{s} + \rho_I + \rho_D\bar{s}^2\right)e^{-\bar{s}\vartheta}} \tag{13}$$

so that the characteristic quasi-polynomial of the control loop is as follows

$$M(\bar{s}) = f\bar{s}^4 + \left(1 + f\lambda^{-1}\right)\bar{s}^3 + \left(\lambda^{-1} + f\right)\bar{s}^2 + \bar{s} + \left[-\beta\rho_D\bar{s}^3 + \left(\rho_D - \beta\rho_P\right)\bar{s}^2 + \left(\rho_P - \beta\rho_I\right)\bar{s} + \rho_I\right]e^{-\bar{s}\vartheta} \tag{14}$$

where $\bar{s} = s\sqrt{a_2}$.

3. DOMINANT THREE-POLE PLACEMENT

As in Zítek, Fišer, and Vyhlídal (2013, 2017) a trio of poles, i.e. roots $\bar{s}_1, \bar{s}_2, \bar{s}_3$ of $M(\bar{s}) = 0$, is placed with the request of their dominance by means of the controller gains ρ_P, ρ_D, ρ_I

setting. So these gains are to be found and for their assessment the product $M(\bar{s})\exp(\bar{s}\vartheta)$ is considered instead of $M(\bar{s})$ since the always nonzero $\exp(\bar{s}\vartheta)$ cannot change the spectrum of $M(\bar{s})$ zeros.

The trio of poles $\bar{s}_1, \bar{s}_2, \bar{s}_3$ is considered composed of one complex conjugate pair

$$\bar{s}_{1,2} = (-\delta \pm j)\nu,\ \nu, \delta > 0, \tag{15}$$

and one real root

$$\bar{s}_3 = -\kappa\delta\nu,\ \kappa \ge 1, \tag{16}$$

where δ is the relative damping of oscillations, ν determines their frequency and κ is the ratio of the real coordinates of $\bar{s}_3$ and $\bar{s}_{1,2}$.

Proposition 1. For the class of similar control loops described by (14) the trio of poles (15), (16) is placed by the controller gain setting ρ_P, ρ_D, ρ_I given as follows

$$\rho_P = \det(\mathbf{A}_1)/\det(\mathbf{A}), \tag{17}$$

$$\rho_D = \det(\mathbf{A}_2)/\det(\mathbf{A}), \tag{18}$$

$$\rho_I = \det(\mathbf{A}_3)/\det(\mathbf{A}), \tag{19}$$

where

$$\mathbf{A} = \begin{bmatrix} \delta\nu + \beta\nu^2(\delta^2 - 1) & -\nu^2(\delta^2 - 1) - \beta\nu^3\delta(\delta^2 - 3) & -1 - \beta\delta\nu \\ -\nu - 2\beta\delta\nu^2 & 2\delta\nu^2 + \beta\nu^3(3\delta^2 - 1) & \beta\nu \\ \kappa\delta\nu + \beta\kappa^2\delta^2\nu^2 & -\kappa^2\delta^2\nu^2 - \beta\kappa^3\delta^3\nu^3 & -1 - \beta\kappa\delta\nu \end{bmatrix}, \tag{20}$$

$$\mathbf{A}_1 = \begin{bmatrix} & -\nu^2(\delta^2 - 1) - \beta\nu^3\delta(\delta^2 - 3), & -1 - \beta\delta\nu \\ \mathbf{B}, & 2\delta\nu^2 + \beta\nu^3(3\delta^2 - 1), & \beta\nu \\ & -\kappa^2\delta^2\nu^2 - \beta\kappa^3\delta^3\nu^3, & -1 - \beta\kappa\delta\nu \end{bmatrix}, \tag{21}$$

$$\mathbf{A}_2 = \begin{bmatrix} \delta\nu + \beta\nu^2(\delta^2 - 1), & & -1 - \beta\delta\nu \\ -\nu - 2\beta\delta\nu^2, & \mathbf{B}, & \beta\nu \\ \kappa\delta\nu + \beta\kappa^2\delta^2\nu^2, & & -1 - \beta\kappa\delta\nu \end{bmatrix}, \tag{22}$$

$$\mathbf{A}_3 = \begin{bmatrix} \delta\nu + \beta\nu^2(\delta^2 - 1), & -\nu^2(\delta^2 - 1) - \beta\nu^3\delta(\delta^2 - 3), & \\ -\nu - 2\beta\delta\nu^2, & 2\delta\nu^2 + \beta\nu^3(3\delta^2 - 1), & \mathbf{B} \\ \kappa\delta\nu + \beta\kappa^2\delta^2\nu^2, & -\kappa^2\delta^2\nu^2 - \beta\kappa^3\delta^3\nu^3, & \end{bmatrix} \tag{23}$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_f f = \left[B_1 + B_1^f f \quad B_2 + B_2^f f \quad B_3 + B_3^f f \right]^T, \tag{24}$$

$$\begin{aligned} B_1 &= b_R \cos(\vartheta\nu) - b_I \sin(\vartheta\nu) \\ B_2 &= b_I \cos(\vartheta\nu) + b_R \sin(\vartheta\nu) \\ B_3 &= e^{-\kappa\delta\vartheta\nu}\kappa\delta\nu\left(-(\kappa\delta\nu)^2 + \lambda^{-1}\kappa\delta\nu - 1\right) \\ B_1^f &= b_R^f \cos(\vartheta\nu) - b_I^f \sin(\vartheta\nu) \\ B_2^f &= b_I^f \cos(\vartheta\nu) + b_R^f \sin(\vartheta\nu) \\ B_3^f &= e^{-\kappa\delta\vartheta\nu}(\kappa\delta\nu)^2\left((\kappa\delta\nu)^2 - \lambda^{-1}\kappa\delta\nu + 1\right), \end{aligned} \tag{25}$$

$$\begin{aligned} b_R &= e^{-\delta\vartheta\nu}\nu\left(\nu^2\delta(3 - \delta^2) + \lambda^{-1}\nu(\delta^2 - 1) - \delta\right) \\ b_I &= e^{-\delta\vartheta\nu}\nu\left(\nu^2(3\delta^2 - 1) - 2\nu\lambda^{-1}\delta + 1\right) \\ b_R^f &= e^{-\delta\vartheta\nu}\nu^2\left(\nu^2(\delta^4 - 6\delta^2 + 1) - \lambda^{-1}\nu\delta(\delta^2 - 3) + \delta^2 - 1\right) \\ b_I^f &= e^{-\delta\vartheta\nu}\nu^2\left(-\nu^2 4\delta(\delta^2 - 1) + \lambda^{-1}\nu(3\delta^2 - 1) - 2\delta\right) \end{aligned} \tag{26}$$

Proof. The trio of poles is placed simultaneously by substituting the complex conjugate pair (15) and the third real pole (16) into the characteristic equation as follows

$$M(\bar{s}_i)\exp(\bar{s}_i\vartheta) = 0, \quad i = 1, 3, \tag{27}$$

where the left-hand side of (28) is obtained after $\bar{s}_i$, $i = 1, 3$, are substituted for $\bar{s}$ in quasi-polynomial (14). Then (27) is arranged gradually for $\bar{s}_i$, $i = 1, 3$, in the way

$$m_L(\bar{s}_i) = m_R(\bar{s}_i), \quad i = 1, 3. \tag{28}$$

The searched gains, ρ_P, ρ_D, ρ_I, appear only on the left-hand side, m_L, and the rest (i.e. the known parameters), including constant *f*, are on the right-hand side, m_R. Thus, in case $\bar{s}_1$

$$m_{L,1} = \tag{29}$$
$$-\begin{pmatrix} \nu^2\left(\delta^2 - 1 + \beta\nu\delta(\delta^2 - 3) + \left(-\beta\nu(3\delta^2 - 1) - 2\delta\right)j\right)\rho_D + \cdots \\ \nu\left(-\delta - \beta\nu(\delta^2 - 1) + (1 + 2\beta\delta\nu)j\right)\rho_P + (1 + \beta\delta\nu - \beta\nu j)\rho_I \end{pmatrix}$$

$$m_{R,1} = e^{-\delta\vartheta\nu}\left[\cos(\vartheta\nu) + j\sin(\vartheta\nu)\right] \times$$
$$\begin{bmatrix} f\nu^4\left(\delta^4 - 6\delta^2 + 1 + 4\delta(1 - \delta^2)j\right) + \cdots \\ (1 + f\lambda^{-1})\nu^3\left(3\delta - \delta^3 + (3\delta^2 - 1)j\right) + \cdots \\ (\lambda^{-1} + f)\nu^2(\delta^2 - 1 - 2\delta j) + \nu(-\delta + j) \end{bmatrix} \tag{30}$$

and in case of $\bar{s}_3$

$$m_{L,3} =$$
$$-\begin{bmatrix} \nu^2(\kappa^2\delta^2 + \beta\kappa^3\delta^3\nu)\rho_D + \nu(-\kappa\delta - \beta\kappa^2\delta^2\nu)\rho_P + \cdots \\ (1 + \beta\kappa\delta\nu)\rho_I \end{bmatrix}, \tag{31}$$

$$m_{R,3} = e^{-\kappa\delta\vartheta\nu} \times$$
$$\left[f(\kappa\delta\nu)^4 - (1 + f\lambda^{-1})(\kappa\delta\nu)^3 + (\lambda^{-1} + f)\kappa^2\delta^2\nu^2 - \kappa\delta\nu \right] \tag{32}$$

Set of relations (29-32) can be expressed in matrix form

$$\mathbf{A}\mathbf{P} = \mathbf{B}, \tag{33}$$

where $\mathbf{A}$ results in (20) and $\mathbf{B}$ in (24). After that the elements of $\mathbf{P} = [\rho_P \ \ \rho_D \ \ \rho_I]^T$ are calculated by formulae (17), (18), and (19) when applying Cramer's rule to (33). The proof is finished. ■

The formulae given by (17-19) provide the placement of the trio of poles (15-16) to the similar control loops. However to ensure also the dominance of these poles this dominance can be proved by the argument increment rule (Zítek, Fišer, and Vyhlídal, 2013) and/or quasi-polynomial root finder (Vyhlídal and Zítek, 2009).

In the next section the spectrum of similar control loops described by (14) is analysed with respect to both the guarantee of three-pole dominance and vanishing f.

4. SPECTRUM-BASED ANALYSIS OF THE SIMILAR CONTROL LOOPS

The fourth order of the control loop quasi-polynomial (14) is connected with the presence of filter $F(\bar{s})$. Without filtering, i.e. for $f \to 0$, it represents the ideal PID control of plant (4) and simplifies to only third order form

$$M_{id}(\bar{s}) = (1 - \beta\rho_D e^{-\vartheta\bar{s}})\bar{s}^3 + (\lambda^{-1} + (\rho_D - \beta\rho_P)e^{-\vartheta\bar{s}})\bar{s}^2 + \\ + (1 + (\rho_P - \beta\rho_I)e^{-\vartheta\bar{s}})\bar{s} + \rho_I e^{-\vartheta\bar{s}} \tag{34}$$

where, however, a delay term appears even with the highest power $\bar{s}^3$. Apparently this quasi-polynomial is no longer *retarded* but *neutral* and therefore its stability may be extremely sensitive to even infinitely small changes of τ. To exclude such inadmissible property of control loop the so-called *strong stability* of (34) is to be provided for any practical implementation of this kind of dynamics. The following condition is necessary for the stability and must be then respected in the controller tuning.

Lemma 1. The neutral quasi-polynomial (34) is strongly stable if and only if its associated difference equation

$$y(t) + \beta\rho_D y(t - \vartheta) = 0\ , \tag{35}$$

is stable, i.e. if

$$|\beta\rho_D| < 1\ . \tag{36}$$

Proof. Since the characteristic equation of (35) is $z - \beta\rho_D = 0$ and its root is $z_1 = \beta\rho_D$ the condition is proved.

The strong stability condition (36) is investigated for both aperiodic ($\lambda \le 0.5$) and oscillatory ($\lambda > 0.5$) processes. Particularly a sensitivity to strong stability loss is detected by placing variable natural frequency angle ν up to 125 percent of ultimate frequency angle ν_k. The similarity number of laggardness is considered within the range $\vartheta \in \langle 0.25, 1.5\rangle$ and with fixed β.

4.1 Illustrative example

Consider process (1) with parameters $a_1 = 1.8$ s, $a_2 = 0.7$ s^2, $b_1 = 1.5$ s, $\tau = 1$ s that can be described according to (10) by the following similarity numbers

$$\lambda = 0.4648\ ,\ \vartheta = 1.1952\ ,\ \beta = 1.7928\ . \tag{37}$$

With respect to identified ultimate frequency angle $\nu_k = 0.7728$ an accelerated control loop dynamics is assigned iteratively as follows

$$\bar{s}_{1,2} = (-\delta \pm j)\nu_i,\ \bar{s}_3 = -\kappa\delta\nu_i,\ \ i = 1,2,\ldots,N\ , \tag{38}$$

where $\delta = 0.25$, $\kappa = 1.3$ and $N = 18$. The natural frequency angles are specified

$$\nu_i = \begin{Bmatrix} 0.6955,\ 0.7728,\ 0.85,\ 0.8578,\ 0.8655,\ 0.8733,\cdots \\ 0.881,\ 0.8887,\ 0.8964,\ 0.9042,\ 0.9119,\ 0.9196,\cdots \\ 0.9274,\ 0.9351,\ 0.9428,\ 0.9505,\ 0.9583,\ 0.966 \end{Bmatrix}. \tag{39}$$

In Fig. 1 the rightmost spectrum of the similar control loops is shown when filter time constant $T_f = 0.01$ s, i.e. f = 0.01195. One can see that trios of poles (38) are safely dominant when controlling the process with the filtered PID controller of which gains are evaluated by (17-19). On contrary in Fig. 2 the spectrum of the similar control loops is recorded if no filter is considered, i.e. $f = 0$. As expected a neutral root chain is generated in the rightmost spectrum and when the condition for the strong stabilization is not satisfied the neutral root chain appears in the right-hand side of the complex plane. The neutral root chain in Fig. 2 (marked by black rings) is then given

$$\bar{s}_k^N = (1/\vartheta)\ln(\beta\rho_D) \pm jk\nu_N,\ \ k = 0,1,2,\ldots\ . \tag{40}$$

For moving the neutral root chain close to the imaginary axis the natural frequency angle necessitates to prescribe above 125 percent of the ultimate one with respect to growing relative damping δ and root ratio κ. All the research made in this matter does not justify the pole placement with such a natural frequency assignment, for more details see Zítek, Fišer, and Vyhlídal (2017) and references therein.

In Section 5 the natural frequency angle assigned by the PID controller setting (17-19) is mapped in space of λ, ϑ when the strong stability boundary is reached under the three-pole dominance guarantee (separately from the position of the neutral root chain that is potentially responsible for the dominance loss).

5. STABILITY BOUNDARY MAPPING IN THE SPACE OF SIMILARITY NUMBERS

In this section the results of Example 4.1 are generalized by mapping the strong stability boundary in space of λ, ϑ. The similarity number, β, is considered to be fixed at the same value as in (37). First, in Fig. 3 the ultimate frequency angle is recorded in considered ranges of λ, ϑ. In Fig. 4 the natural frequency angle in relation to the ultimate one is mapped where both the strong stability boundary and the dominant three-pole placement (separate from the influence of root chain position) are reached. From (36) the strong stability boundary is met in practice when

$$\rho_D = 1/\beta = 0.5577\ . \tag{41}$$

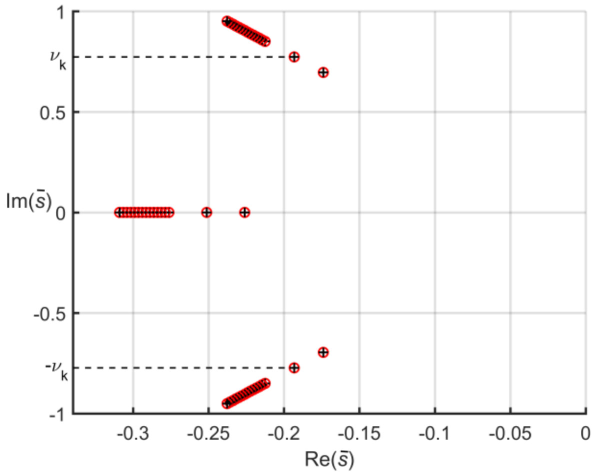

Fig. 1. Rightmost spectrum of (14) (*red rings*) compared with prescribed trios of poles (*black +*)

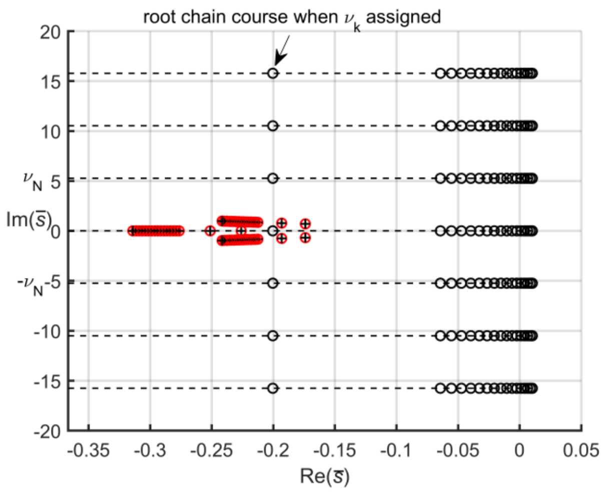

Fig. 2. Rightmost spectrum of (34) and prescribed trios of poles together with neutral root chains

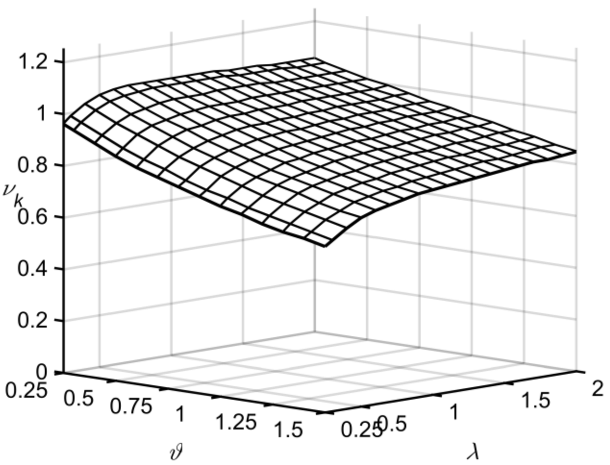

Fig. 3. Mapping of ultimate frequency angle.

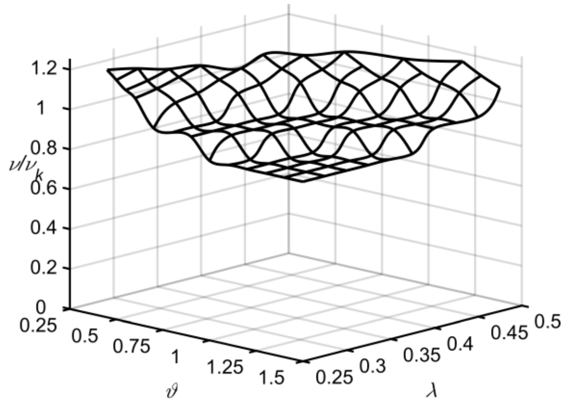

Fig. 4. Ratio of natural frequency angle and ultimate one mapped on the strong stability boundary.

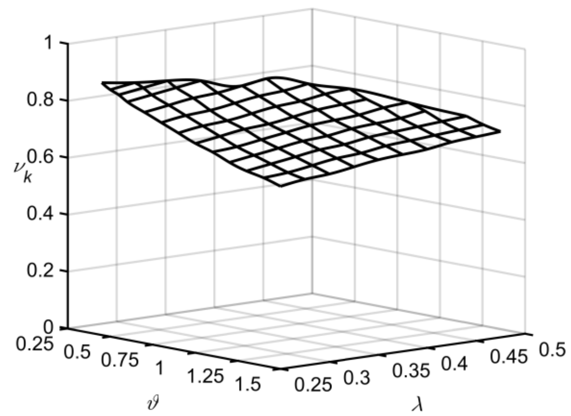

Fig. 5. Detail of Fig. 3 with the ultimate frequency angles considered in Fig. 4.

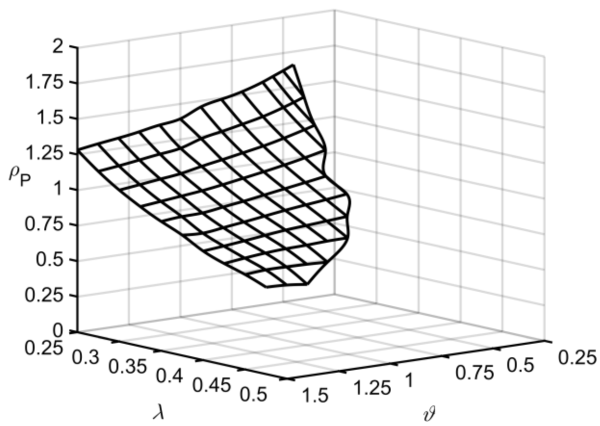

Fig. 6. Proportional gain setting on the strong stability boundary.

In the dominant three-pole placement the relative damping and root ratio are kept fixed at such values to reach the strong stability boundary below 125 percent of the ultimate frequency angle as the assigned one. These values are $\delta = 0.25,\ \kappa = 1.3$. In Fig. 5 only the detail of Fig. 3 is shown

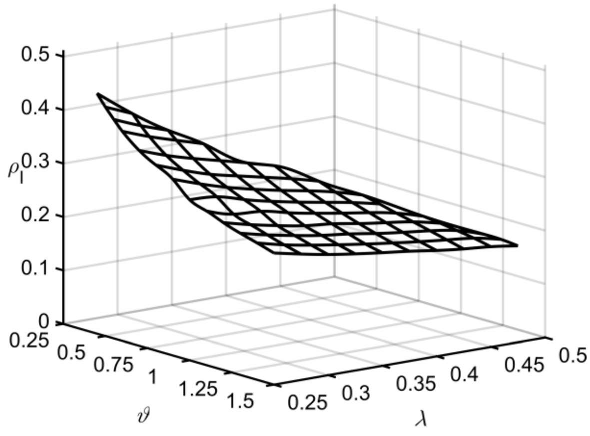

Fig. 7. Integration gain setting on the strong stability boundary.

with the ultimate frequency angles to which are the natural frequency angles related in Fig. 4. In Fig. 6 and 7 the rest of controller gains, ρ_P, ρ_I, settings are recorded leading to the strong stability boundary but preserving separately the three-pole dominance.

Although similarity number β and pole placement parameters δ, κ are assumed constant the mapping of the strong stability boundary is discovering enough the pitfalls of neutral PID control loops.

6. CONCLUSIONS

PID control of non-minimum phase plant (4) is tightly connected with the appearance of a neutral character of the control loop. In terms of setting the controller by means of the dominant pole placement this issue has been systematically investigated for a broad class of considered plants both aperiodic and oscillatory. The strong stability boundary has been found for varying values of prescribed natural frequency of the control loop. It turned out that the most sensitive to the strong stability loss are aperiodic plants. In this respect it is to realize that vanishing filtering necessitates higher than ultimate values of natural frequencies to be prescribed. The comparison of both the schemes results in the envisaged conclusion that the use of filtering requires higher values of ρ_P, ρ_D, ρ_I than the filter-free option for obtaining equivalent control response.

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