Act-and-wait modification of time-delayed feedback control algorithm

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Time-delayed feedback control (TDFC) [1] is a simple and convenient experimental tool for the stabilization of unstable periodic orbits (UPOs) in nonlinear dynamical systems [2]. However, the theory of TDFC is difficult because it deals with infinite-dimensional function space. Since the stability of a controlled UPO is determined by an infinite number of Floquet exponents (FEs), the optimization of the control parameters is a severe problem. A very attractive idea that allows to transform the TDFC into a finite-dimensional controller is to apply an act-and-wait concept [34], which implies a periodic switch on (act) and off (wait) of the delayed feedback term. It has been shown that the time-delayed system can be treated as a finite-dimensional system provided that the duration of waiting is longer than the duration of acting. Here we present an act-and-wait modification of the TDFC for both the nonautonomous [5] and autonomous [6] systems. In our modification, the control gain is periodically switched on and off with the period equal to the double period of the target UPO. For the act-and-wait TDFC, it is necessary to keep constant an appropriate phase difference between the UPO and the act-and-wait switching function. In the nonautonomous case, this is easy to do by using an information of the driving force of the system. In autonomous case, we introduce an additional small-amplitude periodic perturbation in order to lock the phase of the target UPO with that of the act-and-wait switching function.

Due to periodical switching on and off the control perturbation, an infinite-dimensional function space of the TDFC system is reduced to the finite-dimensional state space. As a result, the number of Floquet exponents defining the stability of the controlled orbit remains the same as for the control-free system. The values of these exponents can be effectively manipulated by the variation of control parameters and the very deep minima of the spectral abscissa of FEs can been attained. We demonstrate the advantages of the modification for the chaotic nonautonomous Duffing oscillator and different autonomous systems: the normal form of the subcritical Hopf bifurcation as well as chaotic Lorenz and Rössler systems. We show that the act-and-wait TDFC works in cases were the conventional TDFC fails. In particular, the modified TDFC can stabilize UPOs with an odd number of real unstable FEs using a simple single-input single-output constraint control.

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