Robust PID Controller Design for Plants with Delay Using Similarity Theory

J. Fišer*,#, P. Zítek*, T. Vyhlídal*,#

* *Dept. Instrument. and Control Eng., Faculty of Mechanical Engineering,*
Czech Technical University in Prague, 166 07 Praha 6, Czech Republic
(Tel: +42022435-3953; e-mail: jaromir.fiser@fs.cvut.cz, pavel.zitek@fs.cvut.cz).
Czech Inst. Informatics, Robotics and Cybernetics, Czech Technical University in Prague,
Czech Republic (e-mail: tomas.vyhlidal@fs.cvut.cz).

Abstract: Novel robust PID controller tuning is designed for second-order processes with varying delay in terms of the similarity theory. Similarity numbers are provided to characterize class of dynamically similar control loops enabling to tune all the robust PID controller settings together with a measurement filter adjustment. The novelty of the paper consists in the controller gain and filter constant settings by means of dominant four-pole placement such that the quadruple of placed poles is simultaneously robust. In addition these settings are generalized due to plant's dynamics characteristics expressed by the similarity numbers that describe sets of dynamically similar plants considered stable. For proving the robust stability of the similar control loops an algorithm for the pseudospectral abscissa evaluation is applied. Finally an example of the stability proof is given.

Keywords: PID, robust control, dominant pole placement, similarity theory, pseudospectral abscissa.

1. INTRODUCTION

The robust stability notion related to the ∞-norm was founded by Doyle (1979). Later on in Kwakernaak (1993) a tutorial how to design the robust control based on H_∞-optimization has been presented for plants with general uncertainty models. These models consider uncertainties in both the numerator and denominator of plant's description. Generally, the uncertainties suggested are either structured or unstructured, and more specifically additive or multiplicative (Skogestad and Postlethwaite, 2001). Robust stability assessment based on the uncertainty model knowledge is the key point of properly designed robust controller. There are many works dedicated to the robust controller design where additionally the robust performance is evaluated. This robust design is then based on simultaneous robust stability assessment and robust performance optimization by means of the so-called mixed sensitivity minimization. To mention at least a few of works with consistent approach to this minimization the following ones are given (Dahleh and Pearson, 1988; Kwakernaak 1993; Zhou, Doyle and Glover 1996).

Another way how to design the robust controller utilizes robust pole assignment technique (Åström 1980). In Åström (1980) the robust stability criterion is introduced somewhat differently than it is in Doyle (1979) because this criterion is derived for two-degrees-of-freedom systems bringing about the desired model following. Generally, the robust pole assignment technique is designed to resist the variations in the model describing complex process (Åström 1980). Practically this pole assignment is obtained when the positions of poles placed are restricted to a region which is inside the left half-plane of the complex plane. The region strictly inside the stability region can be of various shapes and then this pole assignment is generalized to regional pole placement (Haddad and Bernstein, 1992). In Wu and Lee (1997) the multi-constraints optimal regional pole placement problem is solved. Frequently the regional pole placement is formulated and solved via an LMI approach (Chilali and Gahinet 1996; Ge, Chiu, and Wang (2002); Henrion, Šebek, and Kučera, 2005; Kosmidou, 2006).

Once the process controlled is with delay the infinite number of poles originates in the control loop and due to finite-dimensional controller considered the dominant pole assignment is to be applied to the controller tuning. Thus only so many poles can be assigned as many controller parameters are available. Establishing the dominance of the pole placement as well as sufficiently separating these poles from the rest of the infinite spectrum only then the pole placement can be properly made, see for instance Ramírez et al. (2017). In case at least four dominant poles are placed the PID controller with filter is successfully tuned if the rest of all the poles are moved spontaneously to the left from the placed rightmost poles in the complex plane (Fišer, Zítek, and Vyhlídal, 2017). Practically, once the goal is to design the robust PID controller inevitably measurement or derivative filter has to be included into the design (Ou, Zhang, and Gu 2006; Goncalves, Palhares, and Takahashi 2008). Major breakthrough in the robust stability theory for time delay systems has been achieved by Kharitonov and Zhabko (1994) with extension of well-known Kharitonov's theorem. This robust stability theory is applied to the robust PID controller tuning in the control loop with the fixed (Fang 2014) and time-varying delay (Wang 2011). Computationally demanding procedures for the robust stability proof are algorithms for pseudospectral abscissa evaluation (Gumussoy and Michiels, 2010; Michiels and Niculescu, 2014; Meerbergen et al., 2017).

The paper is aimed at the robust PID controller tuning based on the dominant four-pole placement and pseudospectral abscissa mapping. To obtain generalized PID controller settings, including filter constant setting, the similarity theory is applied to the delayed control loop description.

2. PROCESS AND UNCERTAIN MODEL DESCRIPTION

Consider stable second-order process with the delay and uncertainties as follows

$$\left(a_2+\Delta a_2\right)\frac{d^2y(t)}{dt^2}+\left(a_1+\Delta a_1\right)\frac{dy(t)}{dt}+y(t)= \\ Ku(t-\tau-\Delta\tau)+Cd(t-\tau-\Delta\tau) \tag{1}$$

where a_1, a_2 are nominal process model coefficients and τ is nominal process time delay. $\Delta a_1, \Delta a_2$ and $\Delta\tau$ are parametric uncertainties corresponding to a_1, a_2 and τ, respectively. K and C are steady-state gains corresponding to the control variable u and disturbance d, respectively. Process model (1) is further simplified applying the similarity theory adopted from Zítek, Fišer, and Vyhlídal (2017). This simplification consists in reducing the number of process model parameters and this model in (2) is derived from that in (1)

$$\left(1+\mu_2\right)\frac{d^2y(\bar t)}{d\bar t^2}+\left(1+\mu_1\right)\lambda^{-1}\frac{dy(\bar t)}{d\bar t}+y(\bar t)= \\ Ku\left(\bar t-\left(1+\mu_\tau\right)\vartheta\right)+Cd\left(\bar t-\left(1+\mu_\tau\right)\vartheta\right) \tag{2}$$

where, (3)

$$\lambda=\frac{\sqrt{a_2}}{a_1},\ \vartheta=\frac{\tau}{\sqrt{a_2}},\ \mu_1=\frac{\Delta a_1}{a_1},\ \mu_2=\frac{\Delta a_2}{a_2},\ \mu_\tau=\frac{\Delta\tau}{\tau},\ \bar t=\frac{t}{\sqrt{a_2}}.$$

λ and ϑ are the similarity numbers called swingability and laggardness, respectively, introduced already in Zítek, Fišer, and Vyhlídal (2013). Novel similarity numbers introduced are

$$\mu_1\lambda^{-1}=\Delta a_1/\sqrt{a_2},\ \mu_\tau\vartheta=\Delta\tau/\sqrt{a_2} \tag{4}$$

which represent percentage changes of the reciprocal swingability similarity number (λ^{-1}) due to uncertainty Δa_1 and the laggardness, ϑ, due to uncertainty $\Delta\tau$. Consecutively, μ_1, μ_2 and μ_τ represent percentage measures of uncertainty in a_1, a_2 and τ. To investigate admissible ranges of μ_1, μ_2, μ_τ for the robust PID controller tuning the process and uncertain model transfer functions are obtained. First to separate the process model transfer function from that of the uncertain model the Laplace transform of (2) in complex variable $\bar s = s\sqrt{a_2}$ is performed under zero initial conditions

$$M(\bar s)Y(\bar s)+\Delta M(\bar s)Y(\bar s)=\Delta N(\bar s)\left[Ke^{-\bar s\vartheta}U(\bar s)+Ce^{-\bar s\vartheta}D(\bar s)\right] \tag{5}$$

where

$$M(\bar s)=\bar s^2+\lambda^{-1}\bar s+1 \tag{6}$$

and

$$\Delta M(\bar s)=\mu_2\bar s^2+\mu_1\lambda^{-1}\bar s,\ \Delta N(\bar s)=e^{-\bar s\mu_\tau\vartheta}. \tag{7}$$

Dividing (5) by nominal characteristic polynomial (6) one obtains

$$Y(\bar s)=G(\bar s)\frac{\Delta_N(\bar s)}{1+\Delta_M(\bar s)}\left[KU(\bar s)+CD(\bar s)\right] \tag{8}$$

where

$$G(\bar s)=e^{-\bar s\vartheta}/M(\bar s) \tag{9}$$

and

$$\Delta_N(\bar s)=\frac{e^{-\bar s\mu_\tau\vartheta}}{\bar s^2+\lambda^{-1}\bar s+1},\ \Delta_M(\bar s)=\frac{\mu_2\bar s^2+\mu_1\lambda^{-1}\bar s}{\bar s^2+\lambda^{-1}\bar s+1}. \tag{10}$$

$\Delta_N(\cdot)$ and $\Delta_M(\cdot)$ are multiplicative numerator and denominator uncertainty, respectively. $\Delta_M(\cdot)$ transfers the spectrum of $M(\bar s)$ zeros to the spectrum of roots in $\bar s$ satisfying the following equation

$$\left(1+\mu_2\right)\bar s^2+\left(1+\mu_1\right)\lambda^{-1}\bar s+1=0. \tag{11}$$

The following pair of roots is obtained from (11) as follows

$$\bar s_{1,2}=\frac{1}{2\lambda}\frac{1+\mu_1}{1+\mu_2}\left[-1\pm j\sqrt{4\lambda^2\frac{1+\mu_2}{\left(1+\mu_1\right)^2}-1}\right] \tag{12}$$

then it is necessarily excluding $\mu_1\neq-1$ and $\mu_2\neq-1$. The former is required by the stability assumption of (1) and the latter by the regularity condition of polynomial (Kharitonov and Zhabko 1994). Admissible ranges of μ_1, μ_2 and also μ_τ, without any loss of generality, then result in

$$\mu_1\in\left\langle\mu_1^-\ \ \mu_1^+\right\rangle,\mu_2\in\left\langle\mu_2^-\ \ \mu_2^+\right\rangle,\mu_\tau\in\left\langle\mu_\tau^-\ \ \mu_\tau^+\right\rangle, \\ -1<\mu_{1,2,\tau}^-<0,\ 0<\mu_{1,2,\tau}^+<1. \tag{13}$$

$\mu_{1,2,\tau}^-$, $\mu_{1,2,\tau}^+$ are lower and upper bounds of parameter uncertainties. In other words these bounds represent the percentage changes of the nominal parameters approaching either -100 or +100 %. However, in practice, these percentage changes are limited either down or up to less than 50 % (Chen and Seborg, 2002). In case of the symmetric parametric uncertainties it results from (13)

$$\mu_{1,2,\tau}\in\left\langle-\mu,\ +\mu\right\rangle,\ 0<\mu<1. \tag{14}$$

Similarly to s also frequency ω is transformed by applying the similarity theory to (1). ω is transformed into the frequency angle defined as $\nu=\omega\sqrt{a_2}$. Applying (14) in (12) and considering all the uncertainty bounds with the same sign then only damped natural frequency angle is changed by non-zero uncertainty μ as follows

$$\nu=\frac{1}{2\lambda}\sqrt{4\frac{\lambda^2}{1\pm\mu}-1}. \tag{15}$$

After that the process is critically damped when $\lambda = \sqrt{1 \pm \mu}/2$. Thus the sign minus increases the range of λ in which the process is oscillatory and vice versa. As regards ϑ , a growth of μ_τ leads to more delayed process (2) and vice versa.

In (2) no change in gains *K* and *C* is considered because the former is absorbed by the controller gains and the latter does not take effect on disturbance response damping as apparent from the next subsection.

1.1 Nominal similar control loops

In practice indispensable part of the PID controller is a filter allowing the robustness achievement. Consider process (8) free of uncertainties, $\Delta_N(\cdot) = \Delta_M(\cdot) = 0$, to be controlled by the PID controller in Laplace transform (Zítek, Fišer, and Vyhlídal, 2017)

$$R(\bar{s}) = \left(\rho_P \bar{s} + \rho_I + \rho_D \bar{s}^2\right)\big/\bar{s} = U(\bar{s})\big/E_f(\bar{s}) \qquad (16)$$

with the control error (e) filtered by

$$F(\bar{s}) = 1\big/(\phi\bar{s} + 1) = E_f(\bar{s})\big/E(\bar{s}) . \qquad (17)$$

In (16) $\rho_P = Kr_P$, $\rho_D = Kr_D / \sqrt{a_2}$, and $\rho_I = Kr_I\sqrt{a_2}$, see Zítek, Fišer, and Vyhlídal (2017). These gains are the dimensionless proportional, derivative and integration gains, respectively, absorbing *K*. In (17) ϕ is the dimensionless filter time constant given as $\phi = T_f / \sqrt{a_2}$, $\phi > 0$, where T_f is the dimensional time constant. The disturbance transfer function of the similar control loops is then obtained using (8), (9), (16) and (17) as $T(\bar{s}) = CG(\bar{s})\big/(1 + G(\bar{s})R(\bar{s})F(\bar{s}))$. This is expressed as

$$T(\bar{s}) = \frac{C\left(\phi\bar{s} + 1\right)\bar{s}e^{-\bar{s}\vartheta}}{\bar{s}\left(\bar{s}^2 + \lambda^{-1}\bar{s} + 1\right)\left(\phi\bar{s} + 1\right) + \left(\rho_P\bar{s} + \rho_I + \rho_D\bar{s}^2\right)e^{-\bar{s}\vartheta}} \qquad (18)$$

from where the denominator constituting the characteristic quasi-polynomial is modified after multiplying by non-zero $\exp(\bar{s}\vartheta)$ to the form

$$Q(\bar{s}) = e^{\bar{s}\vartheta}\left[\phi\bar{s}^4 + \left(1 + \phi\lambda^{-1}\right)\bar{s}^3 + \left(\lambda^{-1} + \phi\right)\bar{s}^2 + \bar{s}\right] + \rho_D\bar{s}^2 + \rho_P\bar{s} + \rho_I . \qquad (19)$$

This modification does not change the spectrum of zeros. For the robust tuning of the PID controller gains including the filter time constant, $\rho_P, \rho_D, \rho_I, \phi$, the dominant four-pole placement is introduced in the next section.

3. DOMINANT FOUR-POLE PLACEMENT

The restricted robust stability region for the dominant four-pole placement is adapted from Chilali and Gahinet (1996) in the way

$$110° < \arg(\bar{s}) < 250°, \; \operatorname{Re}(\bar{s}) < -\delta\nu < 0, \; \nu > \nu_k > 0 \qquad (20)$$

where ν_k is the ultimate frequency angle and δ is the relative damping. The ultimate frequency angle belonging to dimensionless process model (8) free of any uncertainty, i.e. $\mu_1 = \mu_2 = \mu_\tau = 0$, is adopted from Zítek, Fišer, and Vyhlídal (2017) for considered values of similarity numbers λ, ϑ . Region (20) warrants the well damped and fast enough quadruple of poles of the similar control loops described by (19).

Theorem 1. The following four poles

$$\bar{s}_{1,2} = (-\delta \pm j)\nu \, , \; \bar{s}_3 = -\kappa\delta\nu , \; \kappa \ge 1 \, , \; \bar{s}_4 = k\bar{s}_3 , \; k > 1 \qquad (21)$$

are placed in the similar control loops given by (19) if the controller gains ρ_P, ρ_D, ρ_I and filter constant ϕ arranged in the vector

$$\mathbf{P} = \left[\rho_P \;\; \rho_D \;\; \rho_I \;\; \phi\right]^T \qquad (22)$$

are given as follows

$$\mathbf{P}(i) = \det(\mathbf{A}_i)\big/\det(\mathbf{A}), \; i = 1,2,3,4 \, , \qquad (23)$$

where

$$\mathbf{A} = \begin{bmatrix} \delta\nu, & -\nu^2\left(\delta^2 - 1\right), & -1, & -A_R^\phi \\ -\nu, & 2\delta\nu^2, & 0, & -A_I^\phi \\ \kappa\delta\nu, & -\kappa^2\delta^2\nu^2, & -1, & -A_3^\phi \\ k\kappa\delta\nu, & -k^2\kappa^2\delta^2\nu^2, & -1, & -A_4^\phi \end{bmatrix}, \qquad (24)$$

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{B}, & \begin{matrix} -\nu^2\left(\delta^2 - 1\right), & -1, & -A_R^\phi \\ 2\delta\nu^2, & 0, & -A_I^\phi \\ -\kappa^2\delta^2\nu^2, & -1, & -A_3^\phi \\ -k^2\kappa^2\delta^2\nu^2, & -1, & -A_4^\phi \end{matrix} \end{bmatrix}, \qquad (25)$$

$$\mathbf{A}_2 = \begin{bmatrix} \begin{matrix} \delta\nu, \\ -\nu, \\ \kappa\delta\nu, \\ k\kappa\delta\nu, \end{matrix} & \mathbf{B}, & \begin{matrix} -1, & -A_R^\phi \\ 0, & -A_I^\phi \\ -1, & -A_3^\phi \\ -1, & -A_4^\phi \end{matrix} \end{bmatrix}, \qquad (26)$$

$$\mathbf{A}_3 = \begin{bmatrix} \begin{matrix} \delta\nu, & -\nu^2\left(\delta^2 - 1\right), \\ -\nu, & 2\delta\nu^2, \\ \kappa\delta\nu, & -\kappa^2\delta^2\nu^2, \\ k\kappa\delta\nu, & -k^2\kappa^2\delta^2\nu^2, \end{matrix} & \mathbf{B}, & \begin{matrix} -A_R^\phi \\ -A_I^\phi \\ -A_3^\phi \\ -A_4^\phi \end{matrix} \end{bmatrix}, \qquad (27)$$

$$\mathbf{A}_4 = \begin{bmatrix} \begin{matrix} \delta\nu, & -\nu^2\left(\delta^2 - 1\right), & -1, \\ -\nu, & 2\delta\nu^2, & 0, \\ \kappa\delta\nu, & -\kappa^2\delta^2\nu^2, & -1, \\ k\kappa\delta\nu, & -k^2\kappa^2\delta^2\nu^2, & -1 \end{matrix} & \mathbf{B} \end{bmatrix}, \qquad (28)$$

$$\mathbf{B} = \left[B_R \;\; B_I \;\; B_3 \;\; B_4\right]^T , \qquad (29)$$

and

$$\begin{aligned} A_R^\phi &= a_R^\phi \cos(\vartheta\nu) - a_I^\phi \sin(\vartheta\nu) \\ A_I^\phi &= a_I^\phi \cos(\vartheta\nu) + a_R^\phi \sin(\vartheta\nu) \end{aligned} ,$$

$$A_3^{\phi} = e^{-\kappa\delta\vartheta\nu}\left(\kappa\delta\nu\right)^2\left(\left(\kappa\delta\nu\right)^2 - \lambda^{-1}\kappa\delta\nu + 1\right)$$
$$A_4^{\phi} = e^{-k\kappa\delta\vartheta\nu}\left(k\kappa\delta\nu\right)^2\left(\left(k\kappa\delta\nu\right)^2 - \lambda^{-1}k\kappa\delta\nu + 1\right)$$
$$a_R^{\phi} = e^{-\delta\vartheta\nu}\nu^2\left(\nu^2\left(\delta^4 - 6\delta^2 + 1\right) - \lambda^{-1}\nu\delta\left(\delta^2 - 3\right) + \delta^2 - 1\right), \quad (30)$$
$$a_I^{\phi} = e^{-\delta\vartheta\nu}\nu^2\left(-\nu^2 4\delta\left(\delta^2 - 1\right) + \lambda^{-1}\nu\left(3\delta^2 - 1\right) - 2\delta\right)$$

$$B_R = b_R\cos\left(\vartheta\nu\right) - b_I\sin\left(\vartheta\nu\right)$$
$$B_I = b_I\cos\left(\vartheta\nu\right) + b_R\sin\left(\vartheta\nu\right)$$
$$B_3 = e^{-\kappa\delta\vartheta\nu}\kappa\delta\nu\left(-\left(\kappa\delta\nu\right)^2 + \lambda^{-1}\kappa\delta\nu - 1\right)$$
$$B_4 = e^{-k\kappa\delta\vartheta\nu}\kappa\delta\nu\left(-\left(k\kappa\delta\nu\right)^2 + \lambda^{-1}k\kappa\delta\nu - 1\right)$$
$$b_R = e^{-\delta\vartheta\nu}\nu\left(\nu^2\delta\left(3 - \delta^2\right) + \lambda^{-1}\nu\left(\delta^2 - 1\right) - \delta\right)$$
$$b_I = e^{-\delta\vartheta\nu}\nu\left(\nu^2\left(3\delta^2 - 1\right) - \lambda^{-1}\nu 2\delta + 1\right). \quad (31)$$

Proof. The quadruple of poles (21) is placed by substituting the first complex pole from (21) and two real poles from (21) into characteristic equation

$$Q(\bar{s}_i) = 0, \quad i = 1,3,4, \quad (32)$$

where $Q(\bar{s})$ is defined in (19). Thus $\bar{s}_i, i = 1,3,4$ are gradually substituted for $\bar{s}$ in quasi-polynomial (19) and then (32) is arranged as follows

$$q_i^L(\bar{s}_i) = q_i^R(\bar{s}_i), \quad i = 1,3,4. \quad (33)$$

The elements of (22) appear only on the left-hand side, q_i^L, and the rest (i.e. the known parameters), are on the right-hand side, q_i^R. Then, for $\bar{s}_1$ one writes

$$q_1^L = -\left(\begin{array}{c} \nu^2\left(\delta^2 - 1 - 2\delta j\right)\rho_D + \nu\left(-\delta + j\right)\rho_P + \rho_I + \\ e^{-\delta\vartheta\nu}\left[\cos\left(\vartheta\nu\right) + j\sin\left(\vartheta\nu\right)\right]\times \\ \left(\nu^4\left(\delta^4 - 6\delta^2 + 1 + 4\delta\left(1 - \delta^2\right)j\right) + \right. \\ \left.\nu^3\lambda^{-1}\left(3\delta - \delta^3 + \left(3\delta^2 - 1\right)j\right) + \nu^2\left(\delta^2 - 1 - 2\delta j\right)\right)\phi \end{array}\right) (34$$

$$q_1^R = e^{-\delta\vartheta\nu}\left[\cos\left(\vartheta\nu\right) + j\sin\left(\vartheta\nu\right)\right]\times$$
$$\left(\begin{array}{l}\nu^3\left(3\delta - \delta^3 + \left(3\delta^2 - 1\right)j\right) + \nu^2\lambda^{-1}\left(\delta^2 - 1 - 2\delta j\right) + \\ \nu\left(-\delta + j\right)\end{array}\right). \quad (35)$$

and $\bar{s}_3$

$$q_3^L = -\left[\begin{array}{c}\nu^2\kappa^2\delta^2\rho_D - \nu\kappa\delta\rho_P + \rho_I + e^{-\kappa\delta\vartheta\nu}\times \\ \left(\left(\kappa\delta\nu\right)^4 - \lambda^{-1}\left(\kappa\delta\nu\right)^3 + \kappa^2\delta^2\nu^2\right)\phi\end{array}\right], \quad (36)$$

$$q_3^R = e^{-\kappa\delta\vartheta\nu}\times\left[-\left(\kappa\delta\nu\right)^3 + \lambda^{-1}\kappa^2\delta^2\nu^2 - \kappa\delta\nu\right]. \quad (37)$$

In final stage for $\bar{s}_4$ one gets

$$q_4^L = -\left[\nu^2 k^2\kappa^2\delta^2\rho_D - \nu k\kappa\delta\rho_P + \rho_I + \right.$$
$$\left. e^{-k\kappa\delta\vartheta\nu}\times\left(\left(k\kappa\delta\nu\right)^4 - \lambda^{-1}\left(k\kappa\delta\nu\right)^3 + k^2\kappa^2\delta^2\nu^2\right)\phi\right], \quad (38)$$

$$q_3^R = e^{-k\kappa\delta\vartheta\nu}\times\left[-\left(k\kappa\delta\nu\right)^3 + \lambda^{-1}k^2\kappa^2\delta^2\nu^2 - k\kappa\delta\nu\right]. \quad (39)$$

By inspection of relations (34) through (39) and after several simplifications of these relations we obtain the set

$$\mathbf{AP} = \mathbf{B} \quad (40)$$

where $\mathbf{A}$ results in (24) and $\mathbf{B}$ in (29). Finally applying Cramer’s rule to (40) formulae (23) are proved and this closes the proof. ■

The four poles, (21), placed by the setting (23) inside the restricted region (20) can be robust only when these poles become dominant. Thus the four poles are at the same time the rightmost zeros of (19). To check this dominance let be recalled the quasi-polynomial root finder from Vyhlídal and Zítek (2009). Hence only such vector (22) is applicable if the four-pole dominance is guaranteed. Additionally how much the dominant four-pole placement achieved is robust is presented in the next section.

4. ROBUST STABILITY PROOF OF PID CONTROLLER SETTINGS

For robust stability assessment the quasi-polynomial (19) is modified when uncertainties (13) take place in the process model description

$$Q_\Delta(\bar{s}) = e^{\bar{s}\vartheta}\left[\alpha_1\left(\bar{s}\right)\cdot\bar{s} + \alpha_2\left(\bar{s}\right)\cdot\bar{s}^2 + \alpha_3\left(\bar{s}\right)\cdot\bar{s}^3 + \alpha_4\left(\bar{s}\right)\cdot\bar{s}^4\right] +$$
$$\rho_D\bar{s}^2 + \rho_P\bar{s} + \rho_I \quad (41)$$

where

$$\alpha_i\left(\bar{s}\right) = \alpha_i^* e^{\bar{s}\mu_\tau\vartheta}, \; i = 1,2,3,4 \quad (42)$$

and

$$\alpha_1^* = 1, \; \alpha_2^* = \phi + \left(1 + \mu_1\right)\lambda^{-1}, \; \alpha_3^* = 1 + \mu_2 + \left(1 + \mu_1\right)\phi\lambda^{-1},$$
$$\alpha_4^* = \left(1 + \mu_2\right)\phi. \quad (43)$$

Then applying the pseudospectral abscissa appropriately adapted from Michiels and Niculescu (2014) the rightmost point of pseudospectrum of quasi-polynomial (41) is defined

$$\sigma = \max\left\{\mathrm{Re}\left(\bar{s}\right)\middle|\bar{s}\in C : Q_\Delta(\bar{s}) = 0\right\}. \quad (44)$$

The settings computed by (23) are proved to be robust from both rigorous and practical points of view in the sequel.

Theorem 2. The similar control loops characterized by quasi-polynomial (19), on which interval uncertainties (14) are imposed, are robustly stable if in modified relations (42-43)

$$\alpha_i\left(\bar{s}\right) = \alpha_i^* e^{\bar{s}\mu\vartheta}, \; i = 1,2,3,4 \quad (45)$$

$$\alpha_2^* = \phi + \left(1 - \mu\right)\lambda^{-1}, \; \alpha_3^* = 1 + \mu + \left(1 - \mu\right)\phi\lambda^{-1}, \; \alpha_4^* = \left(1 + \mu\right)\phi, \quad (46)$$

$0 < \mu < 1$ is applied and according to (44) all the evaluated σ fulfil the condition

$$\sigma < -\varepsilon, \; \varepsilon > 0. \quad (47)$$

Proof. First determine for any non-zero λ and ϑ the worst case of the uncertainties (13), obeying the rule for the worst case sinusoidal input (Zhou, Doyle and Glover 1996). Since the process damping ratio results from (12) as follows

$$\xi = \frac{1}{2\lambda}\frac{1+\mu_1}{\sqrt{1+\mu_2}} \tag{48}$$

the worst case of process to control is with the poorest damping, except $\xi \to 0$, i.e. $\mu_1 = -1$, see discussion below (12). The lower value of μ_1 and the greater value of μ_2 are the lower results value of ξ. Additionally, with respect to the first similarity number in (4) the lower value of μ_1 and the greater value of λ are the lower results value of ξ. Regarding the either similarity number in (4) the greater μ_τ is the more delayed and thus worse controllable by controller (16) results the process. Hence the worst case of uncertainties considered symmetric as in (14) is derived from

$$\mu_1 = -\mu,\ \mu_2 = \mu_\tau = \mu,\ 0 < \mu < 1 \tag{49}$$

when $\mu \to 1$, again except $\mu_1 = -1$. Substituting (49) into (42) and (43) the relations (45) and (46) are obtained. After substituting (45), (46) and elements of (23) into (41) the pseudospectral abscissa of (41) is computed for varying μ as follows

$$\sigma = \max_{0<\mu<1}\max_{\bar{s}_j \in C}\left\{\mathrm{Re}\left(\bar{s}_j\right), j = 1,2,\ldots\right\}. \tag{50}$$

As far as σ values evaluated by (50) satisfy condition (47) for certain $\varepsilon > 0$ the similar control loops are robustly stable. The Theorem 2 is proved. ■

In other words according to (50) the global rightmost point of the pseudospectra is found out. Efficiently the quasi-polynomial root finder from Vyhlídal and Zítek (2009) is used for this purpose. In contrary to (51) also the effect of small laggardness, i.e. $\mu_\tau = -\mu,\ 0 < \mu < 1$, analogously to the effect of small delays (Vyhlídal et al. 2009), under vanishing ϕ should be tested on the worst case but this is not the case. In practice $\mu_{\max}$ bounding interval (14) as follows

$$0 < \mu < \mu_{\max},\ \mu_{\max} < 1 \tag{51}$$

is identified by (50) when certain exponential decay rate ε required by condition (47) is not satisfied yet. $\mu_{\max}$ shows at the same time to which extent the dominant four-pole placement is robust. This is much more enlightened in the following example.

5. EXAMPLE OF ROBUST STABILITY PROOF

Consider process (2) with the following similarity numbers $\lambda = 1.414$, $\vartheta = 0.265$ and assessed ultimate frequency angle $\nu_k = 1.856$. The dominant four-pole placement in the restricted region (20) is given as $\bar{s}_{1,2} = -0.903 \pm j2.581$, $\bar{s}_3 = -1.174$, $\bar{s}_4 = -2.936$ so that $\delta = 0.35$, $\kappa = 1.3$ and $k = 2.5$. Formulae (23) give off the setting

$$\rho_P = 4.05,\ \rho_D = 2.15,\ \rho_I = 3.1 \text{ and } \phi = 0.015. \tag{52}$$

As shown in Fig. 1 the four-pole dominance is guaranteed by the found rightmost spectrum of (19), separated enough from the rest of infinite spectrum, applying the quasi-polynomial root finder from Vyhlídal and Zítek (2009). In Fig. 2 the robust stability due to (49) for the controller setting (52) is shown with $\mu_{\max} = 0.325$ and $\varepsilon = 0.1$. For usual uncertainty $\mu = 0.25$ $\sigma = -0.22$, i.e. $\varepsilon = 0.2$, results. In practice the robust stability condition (47) is not reached in rigorous manner, i.e. $\mu_{\max} = 1$. To reach a higher value of $\mu_{\max}$ the following dominant four-pole placement is made, $\bar{s}_{1,2} = -1.3 \pm j3.25$, $\bar{s}_3 = -1.3$, $\bar{s}_4 = -1.56$, characterized by ratios $\delta = 0.4$, $\kappa = 1$ and $k = 1.2$.

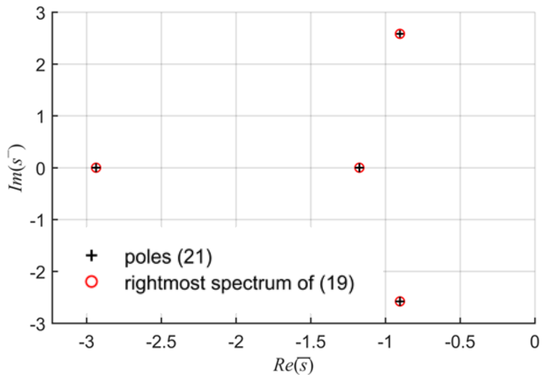

Fig. 1. The rightmost spectrum of (19) compared with the prescribed poles (21)

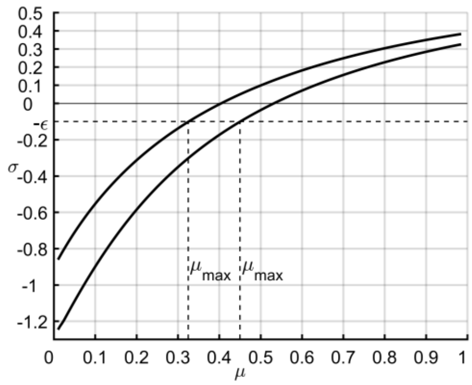

Fig. 2. Pseudospectral abscissa evaluated according to (50)

Formulae (23) provide then the setting

$$\rho_P = 4.377,\ \rho_D = 2.568,\ \rho_I = 2.978 \text{ and } \phi = 10^{-3}. \tag{53}$$

In Fig. 2 the curve of the pseudospectral abscissa corresponds to (53) with $\mu_{\max}$ resulting in 0.45. In Fig. 3 the disturbance rejection with undershoots results not only with worse robustness but also IAE than either one obtained by control with the setting (53).

6. CONCLUSIONS

In summary the robust tuning of the PID controller requires meeting two conditions, namely the four-pole dominance and robust stability (47). As a tool for proving the robust stability the pseudospectral abscissa is selected that makes practically

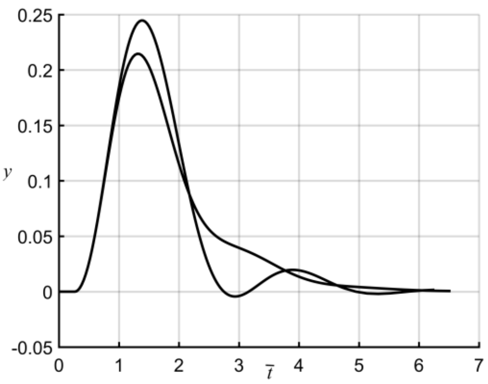

Fig. 3. Disturbance rejections with (54) and (55) settings

the robustness mapping. In other words the higher value of $\mu_{\max}$ results the more robust is the tuning of the PID controller with filter.

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