Necessary and Sufficient Conditions For The Stability Of Uncertain Input-Delayed Systems

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Abstract:
This paper discusses stability analysis supply chain dynamics using feedback control law structure. The case study concerns the inventory control system which is considered as an input-delay system under uncertainties on customer demands with constraints related to losses of stored products. Due to the lead time of the control law and factors such as the customer demand which is supposed to be unknown, the objective is to define a control law which permits to satisfy the end-customer demand and for which the production system requirements will be completely met. The end customer demand is considered as the external perturbation. To study the stability analysis, two types of control law are proposed, both based on a feedback predictor structure. The necessary and sufficient conditions on the existence of control law are then formulated. The results demonstrate that it possible to improve the performances of the supply chain by choosing optimally the control parameters and the specifications of the production system.

Keywords: Time-delay systems, input uncertainties, supply chain, variability of customer demand, stability analysis, inventory control, predictor-feedback structure.

1. INTRODUCTION

In any supply chain, the production orders are issued for the needed products to be purchased and those goods or products are received after a delay named a lead time. Since the delay is encountered in various production systems, the dynamic behavior of many physical processes inherently contains time delays and uncertainties. In addition, time delays are often the main cause of the instability of control systems. For that, there has been increasing interest in research into robust stabilization for uncertain time-delay systems. Application of control engineering to production and inventory control was first studied by (Simon, 1952) by using Laplace Transform to analyze a supply line dynamics. After, further research works have been developed such as (Wang et al., 1987), (Kharitonov, 1998), (Moon et al., 2001), (Dion et al., 2001), (Chiasson and Loiseau, 2007), (Tarbouriech et al., 2011), (Forrester, 1973), (Riddalls and Bennett, 2002), (Ignaciuk and Bartoszewicz, 2011) , (Wang et al., 2012), in which the production system was modeled using block diagrams and controlled through feedback structure. In particular in (Sterman, 1989), the author developed method of interpreting the causal loop diagrams to translate the information flows in the form of bock diagram presentation. This presentation was very useful because that facilitated the use of control theory to analyze such delayed systems.

In this paper, we are interested on the inventory regulation problem in production systems which must respond to the customer demand. We suppose that the customer demand is unknown but bounded by a defined value. Also, the production system is characterized by the presence of delay due to the process time and the products are perishable with fixed preemption rate. Furthermore, positive constraints due to the specifications of the supply chain, such as production and storage capacities, are imposed.

To resolve such problems, different frameworks were proposed based on optimization procedures using programming techniques, empirical experiences and control theory methods as explained before. Our concern focused on the use of the control theory methods which provide an analytic and formal framework, since such systems can be considered as time-delayed systems, with uncertainties on the customer demand.

To deal with the inventory control problem, we propose a control law based on the feedback-predictor structure. The complexity of this study is the fact that customer demand, which corresponds to a disturbance for our input-output system, is unknown. In addition, the inventory level of stored products decreases proportionally over time due to the expiration of stored products. The objective is to provide necessary and sufficient conditions to obtain a control law which stabilize the inventory level and which must meet all required specifications and constraints.

The paper is organized as follows. In section 2, the inventory control problem with principal variables, assumptions and objective are given. In section 3, we recall the control law used based on predictive and saturated feedback structure, considering the preemption rate of products. In section 4, an inventory control structure is described and the necessary and sufficient conditions on the existence of control law are then expressed. We conclude the paper with discussions of using the proposed approach by simulation examples and give directions for future work.

2. PROBLEM STATEMENT AND METHODOLOGY

2.1 Supply Chain description

In our study, we consider a simple supply chain consisting of a single retailer, a single manufacturer and composed of a storage unit. At any moment t, the retailer receives the products of a manufacturer within a specified time θ and issuing a supply order $u(t)$. Also, a demand $d(t)$ is observed and must be completely satisfied. The storage unit is characterized by the level $y(t)$, the incoming flow i.e. the final products coming from a manufacturer, and the outgoing flow from customers demand $d(t)$, and of course the stock is of limited capacity. In its most basic form, the generic model for the inventory level dynamics is described by the following first order delayed equation:

$$\dot{y}(t) = \begin{cases} u(t-\theta) - d(t) & ,\text{for } t \geq \theta, \\ \varphi(t) - d(t) & ,\text{for } 0 \leq t < \theta. \end{cases} \tag{1}$$

$y(t)$ represents the instantaneous inventory level. $d(t)$ is the instantaneous customer demand, which corresponds to the flow of products leaving the stock at any moment t. In reality, to obtain the products, a non-negligible execution time is necessary, and it is noted by θ. It corresponds to the time needed to complete the finite products, from receiving the production order until obtaining the final products. Thus, $u(t)$ corresponds to the instantaneous production order. These are only available from the instant $t = \theta$, precisely because of this time of production θ. Moreover, the function $\varphi(t)$ corresponds to the production flow for instants t between 0 and θ. It is called the work in process WIP of the delay system.

Furthermore, we are interested in perishable products systems. Such systems are modeled by an expiration rate noted σ. After this change on the dynamics of the stock, the fundamental equation takes the following form:

$$\dot{y}(t) = \begin{cases} -\sigma y(t) + u(t-\theta) - d(t) & ,\text{for } t \geq \theta, \\ -\sigma y(t) + \varphi(t) - d(t) & ,\text{for } 0 \leq t < \theta. \end{cases} \tag{2}$$

The parameter σ represents the static loss factor. We notice the appearance of $-\sigma y(t)$ in the fundamental equation. It shows that the stock is decreasing without the application of any control law, because of the preemption of the perishable products.
This model has been used by Blanchini (1990). He treated the communication networks control using the same model. Similarly, Ignaciuk and Bartoszewicz use the same model (2) in their work (Ignaciuk and Bartoszewicz (2011)), and consider the case of multiple sources, which corresponds to the study of a logistic system with several suppliers.

2.2 Constraints and objectives

In the study of our system, production units $u(t)$ and inventory level $y(t)$ are limited resources, and they can take only non-negative values. They are defined as follows.

- The production level $u(t)$ is limited by a minimum supplying order rate denoted u_m and a maximum supplying order rate u_M.
- The inventory level $y(t)$ is bounded by y_m and y_M which are respectively the minimum and the maximum storage capacity.
- The customer demand $d(t)$ is supposed to be unknown but assumed to be bounded by a minimum and a maximum demand rates denoted respectively d_m and d_M.

The controller should be designed taking into account positive and saturation constraints that are formulated as follows.
For all $t \geq 0$

$$y(t) \in [y_m, y_M], \tag{3}$$

$$u(t) \in [u_m, u_M], \tag{4}$$

and every demand function $d(t)$ must satisfy

$$d(t) \in [d_m, d_M]. \tag{5}$$

The problem in to find a control strategy for the system so that the constraints on $y(t)$ and $u(t)$ already mentioned remain always verified for any arbitrary demand satisfying $d(t) \in [d_m, d_M]$. The main objective consists of defining necessary and sufficient conditions for the existence of an admissible control law $u(t)$.

3. FEEDBACK CONTROL STRATEGY

3.1 Prediction structure

As developed in (Abbou et al., 2015), the proposed approach to control systems with delayed inputs is based on a prediction state feedback principle. This structure permits to stabilize the system and to compensate the delay effects present in the loop. The specifications of the production system are introduced as constraints imposed to the controller, so as to forbid any overruns on the production rates or on the inventory levels, which can cause the saturation of the production unit. The role of the controller is then to keep the production rate and so, the inventory level, as far as possible within their limits.

Using the feedback-predictor structure, also known as model reduction or Arstein reduction **?**, the basic idea of state prediction is to compensate the time delay by generating a control law that use directly the corresponding delay-free system. We denote $z(t)$ the prediction of the future state of the stock level $y(t)$. This prediction is carried out over a time horizon from t to $(t+\theta)$, and is expressed by

$$z(t) = e^{-\sigma\theta} y(t) + \int_{t-\theta}^{t} e^{-\sigma(t-\tau)} u(\tau) d\tau. \tag{6}$$

The prediction expressed by (6) can be written by another approach using (2) in the form

$$z(t) = y(t+\theta) + \int_{t}^{t+\theta} e^{-\sigma(t+\theta-\tau)} d(\tau) d\tau. \tag{7}$$

By time derivation of (6), we obtain the following system

$$\dot{z}(t) = -\sigma z(t) + u(t) - e^{-\sigma\theta} d(t). \tag{8}$$

We note that the derivative equation obtained is expressed without delay. We can therefore apply the invariance theory which is recalled in the next paragraph.

3.2 Application of $\mathscr{D}$-invariance principle

The reduction of Artstein can be expressed by the general form $\dot{z}(t) = f(z(t), u(t), d(t))$, with the interval $Z = [z_m, z_M]$ and the interval of the disturbance $d(t)$, $\mathscr{D} = [d_m, d_M]$. Thus we can apply the $\mathscr{D}$-invariance conditions. So Z is $\mathscr{D}$-invariant for this system if and only if the following conditions are fulfilled.

$$f(z_m, d_M) \geq 0 \tag{9}$$

$$f(z_M, d_m) \leq 0 \tag{10}$$

We deduce the following relations.

- For the minimum value $z(t) = z_m$

$$-\sigma z_m + u(t) - e^{-\sigma\theta} d_M \geq 0 \,, \tag{11}$$

- and for the maximum value $z(t) = z_M$

$$-\sigma z_M + u(t) - e^{-\sigma\theta} d_m \leq 0 \,. \tag{12}$$

We consider two values of the control law, u_1 and u_2 which fulfill the constraint (4) and expressed by

$$u_1 \in [u_m, u_M], \tag{13}$$

and

$$u_2 \in [u_m, u_M]. \tag{14}$$

In addition, we suppose that the interval $[z_m, z_M]$ for the system (2), verify the following condition

$$z_m \leq z_M. \tag{15}$$

By interpreting the concept of $\mathscr{D}$-invariance, and taking into account the inequalities (11) and (12), we suppose

- u_1 verifying (13) such as

$$u_1 \geq \sigma z_m + e^{-\sigma\theta} d_M, \tag{16}$$

- and u_2 verifying (14) such as

$$u_2 \leq \sigma z_M + e^{-\sigma\theta} d_m. \tag{17}$$

The following conditions are deduced from the inequalities (16), (17), (13) and (14).

$$\sigma z_m + e^{-\sigma\theta} d_M \leq u_1 \leq u_M \tag{18}$$

$$u_m \leq u_2 \leq \sigma z_M + e^{-\sigma\theta} d_m \tag{19}$$

Proposition 1. Given the system of form (2), as well as z_m and z_M verifying (15), there exists an affine or a hybrid control law which verifies the constraints (13) and (14), and that the interval $[z_m, z_M]$ is $\mathscr{D}$-invariant for the closed-loop system (8), if and only if the following two conditions are verified.

$$\sigma z_m + e^{-\sigma\theta} d_M \leq u_M \tag{20}$$

$$u_m \leq \sigma z_M + e^{-\sigma\theta} d_m \tag{21}$$

Proof. The inequalities (18) and (19) can be deduced from the inequalities (16) and (17) taking into account (13) and (14). Therefore, if the parameters u_1 and u_2 verify (13) and (14), (16) and (17), then they verify (18) and (19). Inversely, if 18) and (19) are verified, then $u_1 = u_M$ and $u_2 = u_m$ can be chosen. Given the parameters z_m and z_M, this choice define an affine or a hybrid control law, such that the interval $[z_m, z_M]$ is $\mathscr{D}$-invariant for the closed loop system.

4. CONTROL LAW ADMISSIBILITY

4.1 *Proposed types of control laws*

We introduce two forms of control laws $u(t)$ that stabilize the inventory level $y(t)$ of the closed loop dynamic system, taking into account positive and saturation constraints (3) (4). The first control law is affine of feedback-predictor type, while the second one is a bang-bang control law.

Affine control law This type of the control law is an affine one, such as, for all $d(t) \in \mathscr{D}$, and for $z(t) \in \mathscr{Z}$, the affine control law is defined as

$$u(t) = \begin{cases} u_1 & ,\text{for } z(t) = z_m, \\ u_2 & ,\text{for } z(t) = z_M. \end{cases} \tag{22}$$

It is structured as follows.

$$u(t) = \begin{cases} K(z_0 - z(t)) & ,\text{for } u_1 \neq u_2, \\ u_1 = u_2 & ,\text{for } u_1 = u_2. \end{cases} \tag{23}$$

- K is a static gain expressed by $K = \frac{u_1 - u_2}{z_M - z_m}$.
- z_0 is the stock order of the controlled system expressed by $z_0 = \frac{u_1 z_M - u_2 z_m}{u_1 - u_2}$.
- $z(t)$ is the prediction of the future state of the inventory level.

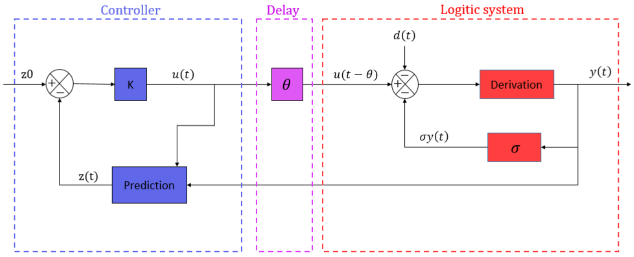

Fig. 1. Closed loop system with affine control law

Bang-bang control law Another type of control laws is used in the study, and it is the way of defining $u(t)$ in the form of a bang-bang control law. This law is expressed as a hybrid system and belongs to the class of well-known optimal control laws. It can take either the minimum value u_2 or the maximum value u_1. It is given by the following expression.

$$u(t) = \begin{cases} u_1 & ,\text{pour } z(t) \leq z_m, \\ u_2 & ,\text{pour } z(t) \geq z_M. \end{cases} \tag{24}$$

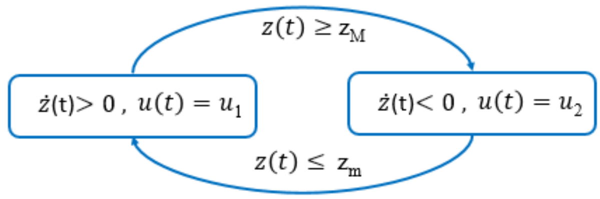

Fig. 2. Bang-bang control automaton

4.2 *Admissibility conditions*

Definition. (Control law admissibility) A control law is admissible if for any initial condition $y(0) \in [y_m, y_M]$, and any WIP $\varphi(\tau)$ having $\tau \in [0, \theta[$, there exists real control parameters u_1, u_2, z_m and z_M such that the unique solution of the closed loop system verifies the constraints on the inventory level $y(t)$ (3) and the production order $u(t)$ (4), for $t \geq 0$ for every customer demand $d(t)$ satisfying (5).

In order to determine the admissibility conditions of the control law of the system (2), we apply the principle of state feedback prediction. So that the expression (7) justifies the term of prediction that we used to denote $z(t)$. This identity also shows that

$$y(t+\theta) = z(t) - \int_t^{t+\theta} e^{-\sigma(t+\theta-\tau)} d(\tau) d\tau. \tag{25}$$

When the system evolves in time, the variable $d(t)$ varies between d_m and d_M and the variable $z(t)$ vary between z_m and z_M. Therefore $y(t)$ will vary between two exact bounds noted y_1 and y_2. We assume in the following work that

$$y_1 \leq y_2. \tag{26}$$

Since the value of $z(t)$ is determined only by the values defined by $d(\tau)$ for the instants preceding t, and the integral depends only on the values taken by $d(\tau)$ for the instants following t, we deduce the relation that exists between the bounds y_1, y_2 and z_m, z_M of the zones traversed by the variables $y(t)$ and $z(t)$.

Proposition 2. With the above notations, the exact values of the reachable output bounds y_1 and y_2 take the following forms:

- for $z(t) = z_m$ and $d(t) = d_M$
$$y_1 = z_m - \frac{1-\mathrm{e}^{-\sigma\theta}}{\sigma} d_M \,, \tag{27}$$
- and for $z(t) = z_M$ and $d(t) = d_m$
$$y_2 = z_M - \frac{1-\mathrm{e}^{-\sigma\theta}}{\sigma} d_m \,. \tag{28}$$

Proof. The integral in the expression (25) is a convolution, whose kernel is stable, its norm in the L_1 sense being $\frac{1-\mathrm{e}^{-\sigma\theta}}{\sigma}$. We deduce that when
$$d_m \le d(t) \le d_M,$$
the integral varies between
$$\frac{1-\mathrm{e}^{-\sigma\theta}}{\sigma} d_m \le \int_t^{t+\theta} e^{-\sigma(t+\theta-\tau)} d(\tau)d\tau \le \frac{1-\mathrm{e}^{-\sigma\theta}}{\sigma} d_M.$$
The right hand side of expression (25) is therefore smaller than the right hand side of expression (28), and larger than the right hand side of expression (27). The equality comes from the fact that the limits y_1 and y_2 are really reachable.

- If $z(t) = z_m$, and the demand $d(t)$ applied between instants t and $t+\theta$ is equal to d_M, then $y(t+\theta)$ assumes the value y_1 verifying (27). This value is therefore the lower bound of the set traversed by $y(t)$ when $t \ge \theta$.
- In the same way, if $z(t) = z_M$, and the demand applied between t and $t+\theta$ is d_m, then we see that $y(t+\theta)$ takes exactly the value y_2, which is therefore the lower bound of the set traversed by $y(t)$ when $t \ge \theta$.

Corollary 1. The system (2) and the prediction (7) being given, and the numbers y_1, y_2 and z_m, z_M verifying (26), (15), (27) and (28) being given, it is observed that the two the following statements are equivalent.
$$\forall\, t \ge 0 \,, \forall\, d(t) \in [d_{min}, d_{max}],\; z(t) \in [z_{min}, z_{max}] \,, \tag{29}$$
$$\forall\, t \ge \theta \,, \forall\, d(t) \in [d_{min}, d_{max}],\; y(t) \in [y_1, y_2] \subset [y_{min}, y_{max}] \,. \tag{30}$$

Proof. From proposition 2 it is clear that (29) implies (30). Inversely, there exists a value of t for which $z(t)$ is not in the interval $[z_m, z_M]$. Two cases occur, depending on whether $z(t)$ is greater than z_M or smaller than z_m.

- In the first case, if $z(t) > z_M$, a demand equal to d_M applied between the instants t and $t+\theta$ causes $y(t+\theta)$ to take a value smaller than y_1.
- In the second case, if $z(t) < z_M$, the demand equal to d_m produces an output greater than y_2, which completes the proof.

5. MAIN RESULTS AND DISCUSSION

From the above results, we can formulate the necessary and sufficient conditions ton obtain a control law as follows.

Proposition 3. Given the system of the form (2), the control law $u(t)$ of affine type (23), or bang-bang type (24) for which the system is stable, is admissible if and only if

- the control parameters u_1, u_2, z_m and z_M verify (16), (17), (13), (14)and(15),
- the output parameters y_1, y_2 verify (26) and (30).

Corollary 2. Given the system of the form (2), there exists u_1 and u_2 such that the control law $u(t)$ is admissible if and only if the parameters z_m and z_M satisfy (20), (21), (13), (14), (15) and $y_m \le y_1$ and $y_2 \le y_M$.

The conditions are:
$$\sigma z_m + e^{-\sigma\theta} d_M \le u_M$$
$$u_m \le \sigma z_M + e^{-\sigma\theta} d_m$$
$$y_m \le z_m - \frac{1-e^{-\sigma\theta}}{\sigma} d_M$$
$$z_M - \frac{1-e^{-\sigma\theta}}{\sigma} d_m \le y_M$$
$$z_m \le z_M$$
These conditions are written in form of inequalities that depend on the parameters θ, σ, z_m and z_M, y_m and y_M, u_m and u_M and d_m and d_M. They are classified in different categories:

- the intrinsic parameters of the system are θ and σ.
- the parameters related to the specification of our system are y_m and y_M, u_m and u_M, d_m and d_M.
- the parameters z_m and z_M are used to determine the control law.

Geometrically

- First, we define the expressions z_a, z_b, z_c and z_d based on the conditions above.
$$z_a = y_m + \frac{1-e^{-\sigma\theta}}{\sigma} d_M$$
$$z_b = \frac{1}{\sigma}(u_M - e^{-\sigma\theta} d_M)$$
$$z_c = \frac{1}{\sigma}(u_m - e^{-\sigma\theta} d_m)$$
$$z_d = y_M + \frac{1-e^{-\sigma\theta}}{\sigma} d_m$$
- After, we define the admissible area of existence of control law in the plan (z_m, z_M). By simple projection in this plane, we can eliminate the control parameters z_m and z_M.
- Referring to (3), the necessary and sufficient conditions of existence of control parameters, z_m and z_M satisfying these conditions, are simplified to $z_a \le z_b$, $z_c \le z_d$ and $z_a \le z_d$.

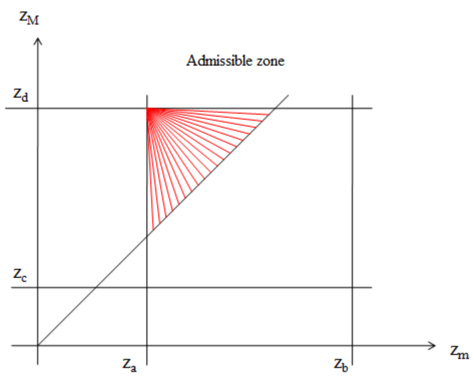

Fig. 3. Illustrative graph for conditions

As a result we obtain:

- necessary and sufficient conditions for admissible control law for $\sigma \neq 0$,

$$\sigma y_m + d_M \leq u_M \tag{31}$$
$$u_m \leq \sigma y_M + d_m \tag{32}$$
$$y_m + \frac{1-e^{-\sigma\theta}}{\sigma} d_M \leq y_M + \frac{1-e^{-\sigma\theta}}{\sigma} d_m. \tag{33}$$

- necessary and sufficient conditions for admissible control law for $\sigma = 0$,

$$\begin{aligned} d_M &\leq u_M \\ y_m + \theta d_M &\leq z_m \\ u_m &\leq d_m \\ z_m &\leq z_M, \end{aligned}$$

which leads to

$$d_M \leq u_M \tag{34}$$
$$u_m \leq d_m \tag{35}$$
$$y_m + \theta d_M \leq y_M + \theta d_m. \tag{36}$$

At the end of this approach, we have obtained the necessary and sufficient conditions for admissible control laws for either affine type or bang-bang type, in the case of perishable final products (31), (32) and (33), and in the general case for any type of final products (34), (35) and (36).

6. ILLUSTRATION EXAMPLES

In order to illustrate the effect of the proposed control strategy, following the theoretical study, we consider in this simulation example the logistic system of the form (2), and we apply either an affine control law or a bang-bang control law.
For this system, we follow a co-design methodology in order to calculate the system parameters, so that the necessary and sufficient conditions of existence given before are all satisfied. We have obtained the values of the system parameters as follow.

- Customer demand $d(t)$: $d_m = 25$, $d_M = 35$.
- Inventory level $y(t)$: $y_m = 0$, $y_M = 85$ with loss rate $\sigma = 0.2$.
- Control law $u(t)$: $u_m = 20$, $u_M = 45$ with the delay $\theta = 6$.
- Prediction interval $Z = [z_m, z_M] = [123, 148]$.
- Control law parameters $u_2 = u_m = 20$, $u_1 = u_M = 45$, $K = 1$ and $z_0 = 168$.
- Initial conditions $y(0) = 50$, $\varphi(t) = 33$, $z(\theta) = 130.22$.

In our study, we apply a random signal form of the customer demand $d(t)$ that evolves arbitrary between d_m and d_M.

Fig. 4. Random demand signal

6.1 Case of an affine control law

The obtained results for the case of an affine control law are described on figures 5 and 6.

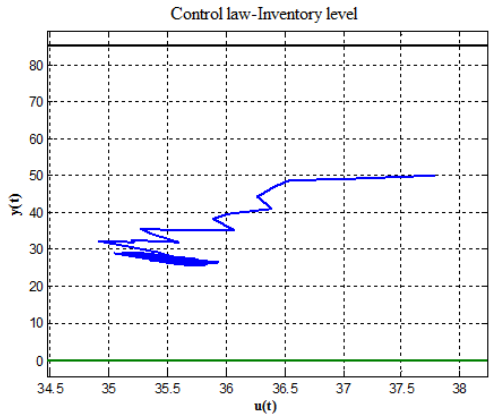

Fig. 5. Trajectory in the plane (u, y)

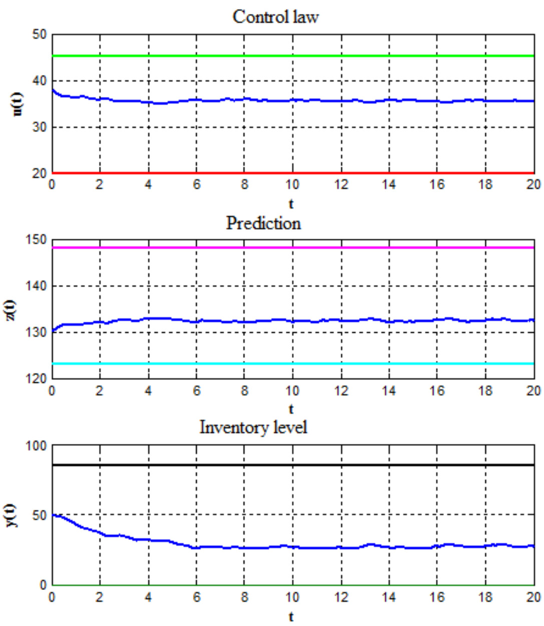

Fig. 6. The temporal variations of $u(t)$, $z(t)$ and $y(t)$

6.2 Case of a bang-bang control law

The obtained results for the case of a bang-bang control law are described on figures 7 and 8.

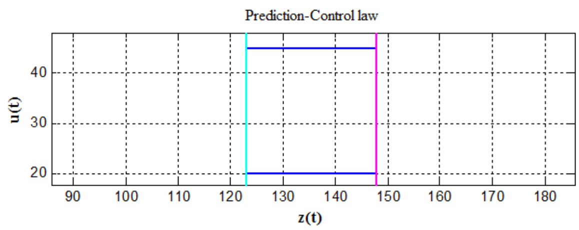

Fig. 7. $(z(t), u(t))$ trajectory

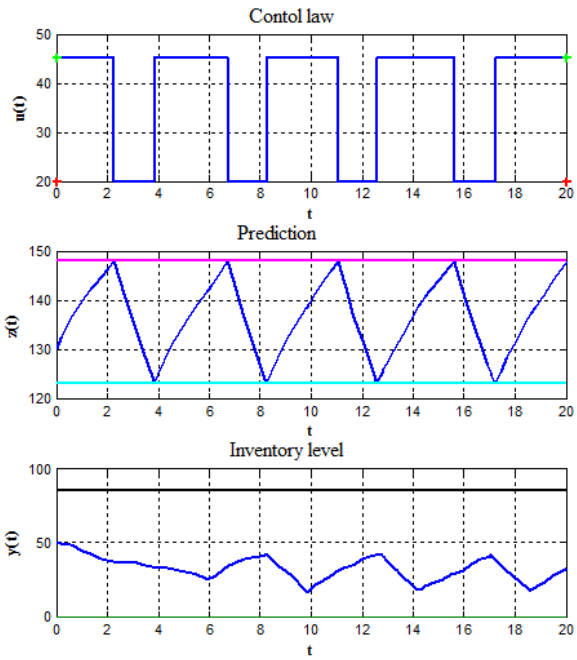

Fig. 8. The temporal variations of $u(t)$, $z(t)$ and $y(t)$

6.3 Simulation analysis

We can say that the inventory level $y(t)$ has no overruns of y_M, and is always positive. The same remark is noted for the control law $u(t)$ which remains always between u_m and u_M. So the positive and saturation constraints (4) (3) are well respected. Moreover, $z(t)$ evolves inside the interval $[z_m, z_M]$, which verify the $\mathscr{D}$-invariance conditions.

In addition, we notice that the evolution of the inventory level $y(t)$ according to the control law $u(t)$ does not show any exceed of the domain limited by the physical constraints of $y(t)$ and $u(t)$, which explain the control law admissibility for every customer demand varying between 25 and 35.

7. CONCLUSION

This paper deals with the problem of perishable inventory control of supply chain, subject to a loss factor σ and production delay θ, using an approach based on control theory. The system is subjected to positive and saturation constraints related to the physical characteristics of the production order $u(t)$ and the inventory level $y(t)$. These constraints must be taken into account in the conception of control strategies for the delayed logistic system in order to satisfy any arbitrary and limited customer demand $d(t)$. More specifically, we presented the delayed dynamic model of the system, on which we have applied Arstein's reduction to compensate the delay and to obtain an equivalent non delayed system. Then we have found the necessary and sufficient conditions for the existence and admissibility of the control laws, in order to stabilize the dynamic system.

In the continuity of this study, several perspectives can be elaborated and developed in further work. First we can assume a variable expiration rate σ as a function of time t, and study its impact on the control laws structures. Similarly, we have considered that the delay θ is constant, it would be interesting to extend this approach in the case of uncertain or variable delays. Moreover, it is necessary to use the approach of our study to deal with the problem of robustness with respect to uncertainty on σ and θ. Finally, this study deals with entirely unknown customer demand $d(t)$. It is necessary to consider an estimated demand $d(t)$ and exploit it in the results and methods developed in this paper.

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