A numerical approach for the stability of large-scale time-periodic time-delay systems

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Broyden's method is a general iterative method commonly used for nonlinear systems of equations, when very little information is available about the problem. We develop an approach based on Broyden's method for nonlinear eigenvalue problems and adapt it to problems stemming from the study of stability of periodic time-delay systems. Our numerical approach is designed for problems where the evaluation of a matrix vector product is computationally expensive. We show that this is the case for a problem appearing in the study of stability of time-periodic time-delay systems $x'(t) = A(t)x(t) + B(t)x(t - \tau)$, where $A(t), B(t) \in \mathbb{R}^{n\times n}$ are periodic and large, i.e., $n \gg 1$. More precisely, we adapt our approach by noting that the characteristic equation of the time-periodic system is a nonlinear eigenvalue problem where the matrix-vector action can be computed by solving an ordinary differential equation. Accurate solution of this differential equation is computationally expensive. The flexibility of our Broyden approach allows us to naturally handle the trade-off between accuracy and computation time. Further improvements of the algorithm is possible by exploiting the structure of the Jacobian matrix and allows us to incorporate it into the algorithm to improve convergence. The algorithm exhibits local superlinear convergence for simple eigenvalues, and we characterize the convergence. Several eigenvalues can be computed in a robust way by applying techniques for deflation for nonlinear eigenvalue problems. A specific problem in machine tool milling, coupled with a partial-differential equation (modeling vibrations in the workpiece) is used to illustrate the approach. The method is successfully applied to problems of dimension $n = 10^4$, and even larger problem when using high-performance computing.