Observer-Based Delayed Resonator with Acceleration Feedback

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Abstract: In this study, a new approach for composing a delayed resonator structure with acceleration feedback is introduced. Classical delayed resonator is modified into an observer-based structure. Performance of the classical and observer-based delayed resonators are investigated on an experimental setup. Tuning parameters of delayed resonators are kept same for all process. Experimental results demonstrate the successful performance of the observer-based delayed resonator.

Keywords: Delayed Resonator, Active Control, Vibration.

1. INTRODUCTION

Vibrations in a mechanism are usually undesired and many researchers developed several techniques to suppress them thorough out the years. One of the common techniques incorporates an auxiliary mass attached to the system which is called dynamic vibration absorber. Dynamic vibration absorbers have a long history, more than a century, first introduced and patented by Frahm (1909). Frahm's invention was a simple design, it only consists of an auxiliary mass which was attached to the main body to alter the resonance frequency of the total system. Later, it was improved by Ormondroyd and Den Hartog (1928) by adding damping to the structure. These methods were the start point for the passive absorption techniques and more studies have been done over the years especially on tunable absorbers which a few of them are reported in Den Hartog (1956), Soong and Dargush (1997), Krenk (2005).

Passive control strategy may be one of the suitable option to suppress the undesired vibration on the primary structure which is excited close to resonance frequency.

On the other hand, active vibration absorbers generate an independent force to eliminate the undesirable effects of the harmonics via actuators. It also ensures a better suppression behaviour for different vibration modes. Active vibration control methods, which are commonly used in aerospace industry, may be classified as integral-based as in Shin (2001), Khot et al. (2012) Jnifene and William (2005), sliding mode as in Li et al.(2013), Hu and Zhu (2012), nonlinear as in Wagg and Neild(2010), Lang et al. (2013) Ahmadabadi and Khadem (2014), and positive position feedback as reported by Fanson and Caughney, (1990), Song et al., (2002), Omidi and Mahmoodi (2014).

However, delayed resonator (DR) may be an ideal alternative for active vibration suppression in these fields. Delayed resonator is actually a resonator that is a modified absorber with proper time delay in the feedback loop introduced by Olgac and Holm-Hansen (1994), Olgac (1995). In the DR structure auxiliary mass is forced to oscillate with the frequency same as the primary mass effected by the disturbance and absorbs the vibrations as a result. In addition, non-complex nature of delayed resonator against extensively used controllers such as PD application on the same systems may be represented as the advantage of this control strategy Elmali et. al (2000). Delayed resonator as a vibration absorber uses partial or full delayed state feedback and offers real-time tunability for every frequency Olgac and Holm-Hansen, (1995). This approach may be based on an unusual control strategy which is called delay scheduling that utilizes time delay as a control parameter Olgac et. al, (2007).

There have been also variety of delayed resonator applications differ with the feedback type that are position, speed, and acceleration. First approach to apply delayed resonator into the vibrating system is stated as delayed control with position feedback in an earlier research of Olgac and Hosek (1997). This work also discussed the selection of the design parameters such as mass, damping, and stiffness of the both primary mass and resonator. Intention of this work is to ensure asymptotic stability of the integrated system along the excitation frequency and desired quick frequency-tuning behavior of delayed resonator.

Speed feedback has been exhibited as another option for delayed resonator application. The delayed resonator with velocity feedback is explored to absorb the torsional vibration Filipović and Olgac, (2002). As distinct from this research, dual frequency fixed delayed resonator (DFFDR) delayed resonator may achieve to resonate in two different frequencies. Therefore, suppressing in both frequencies may be guaranteed differently from classical delayed resonator approach in an earlier one Filipović and Olgac (1997). Furthermore, another unique work comprising experimental tests and completed stability analysis Eris and Ergenc (2016) uses both position and speed feedback as a multiple-delay delayed resonator. Double feedback design also led to extend the operable frequency range reported in a recent work of Eris et.al. (2018)

On the other hand, acceleration feedback may be employed to design delayed resonator as in Olgac et. al. (1997). Delayed acceleration feedback that converts whole system into neutral system whose stability analysis is realized via spectral methods in another paper by Vyhlídal et. al. (2014). A recent work by Pilbauer et. al. (2016) also utilized distributed delay on acceleration feedback. It also eventuated with two advantages which are obtaining more suitable characteristics against neutral behavior with lumped delay and performing like moving average filter on acceleration measurement behavior of distributed delay.

According to all these methodologies and researches, delayed resonator can be introduced as successful and basic control strategy that can accomplish suppressing carbody vibrations. Besides, these workings can be exemplified with the delayed resonator applications on railway vehicles as in Eris et. al. (2014, 2015).

All DR applications, it is considered that the feedback signal is available and accurate as a sensor output. But in some cases sensor output may not be available or not accurate. In this study, observer-based delayed resonator is implemented with acceleration feedback. The classical and observer-based delayed resonator designs are tested on an experimental setup and the performance of both structure are evaluated. As an advantage, observer-based controller design strategy guarantees the generation of accurate control signal without any noise.

In section 2, classical DR is summarized and observer based DR is introduced. In section 3, experimental study is presented and section 4 is the conclusions.

2. DELAYED RESONATOR

2.1 Classical Delayed Resonator

As a dynamic system delayed resonator is attached to a SDOF primary mass and aims absorbing the vibrations on that body (Fig. 1). Dynamic model of the delayed resonator, which is attached to primary mass, with parameters of the structure mass, spring and damping constants (m_a, k_a, b_a) is given in equation (1).

$$m_a\ddot{x}_a(t) + b_a\dot{x}_a(t) + k_a x_a(t) = u(t) \tag{1}$$

Acceleration feedback method is selected to control the delayed resonator and the control signal is derived as in the equation (2)

$$u(t) = g\ddot{x}_a(t-\tau) \tag{2}$$

, where g is the feedback gain and τ is the time delay.

Regarding an approach in an earlier research of Olgac and Holm-Hansen (1994), dominant poles of the delayed resonator are be placed at $s_{1,2} = \pm j\omega$, on the imaginary axis, where ω is the excitation frequency. Therefore, the tuning parameters are computed in equation (3) and equation (4) as follows

$$g = \frac{1}{\omega^2}\sqrt{(b_a\omega)^2 - (m_a\omega^2 - k_a)^2} \tag{3}$$

$$\tau = \frac{1}{\omega}\left(\text{atan}\left(\frac{c_a\omega}{m_a w\omega^2 - k_a}\right) + 2(l-1)\pi\right) \tag{4}$$

, where $l = 1, 2, \ldots$ is the branch of g-τ curves which generates characteristic roots the imaginary axis at $+-j\omega$ Olgac et. al, (1997).

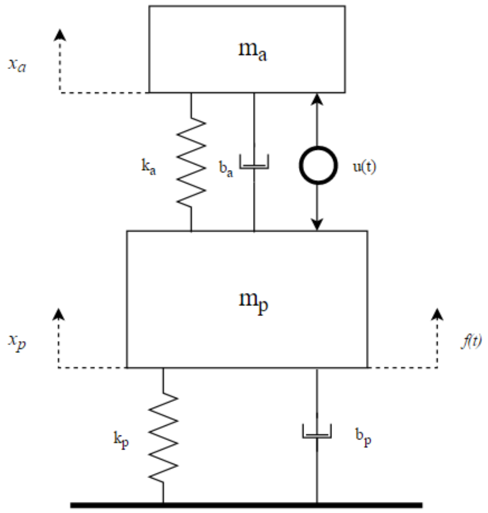

Fig. 1. SDOF Primary Mass with Delayed Resonator

Accordingly, natural frequency and characteristic equation of the delayed resonator are derived in equation (5) and equation (6) as follows

$$\omega = \sqrt{\frac{k_a}{m_a}} \tag{5}$$

$$CE(s) = m_a s^2 + b_a s + k_a s - g s^2 e^{-\tau s} \tag{6}$$

, where s denoted Laplace variable.

A linear time invariant time delayed system, which has a transcendental characteristic equation (6), has infinite number of characteristic roots of which two of them located on imaginary axis by design. The delayed resonator aims to absorb the undesired vibration on primary body that is generated by harmonic disturbance force with the frequency of ω (Fig. 1). Nevertheless, this absorber-like behavior is valid for only a stable system. Notice that, in this study, acceleration feedback is utilized to control they system which introduces time delay term to the highest degree of the differential equation (1). These type of systems are known as neutral systems and their stability has to be checked meticulously. In this study, we utilized the method for determining the imaginary axis eigenvalues of Louisell(2002) and CTCR method of Olgac and Sipahi (2003) for the stability analysis.

The state space representation of the single degree of freedom (SDOF) mass with delayed resonator (Fig. 1) is given as

$$\dot{x}(t) = Ax(t) + Bf(t) + A_t u(t) \tag{7}$$

, where the system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_a}{m_a} & \frac{-b_a}{m_a} & \frac{k_a}{m_a} & \frac{b_a}{m_a} \\ 0 & 0 & 1 & 0 \\ \frac{k_a}{m_p} & \frac{b_a}{m_p} & \frac{-(k_a+k_p)}{m_p} & \frac{-(b_a+b_p)}{m_p} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_p} \end{bmatrix}$$

$$A_t = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{g}{m_a} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{-g}{m_p} & 0 & 0 \end{bmatrix}$$

The overall system operates under partial feedback (delayed acceleration). Acceleration feedback is problematic especially in existence of noise in the measurement system. Thus an observer is designed to estimate the states of the delayed resonator during the operation

2.2 Observer-Based Delayed Resonator

Observers are dynamic structures and they are designed to estimate all states or some of states of a system. Observers are utilized especially when there is no system available to measure certain states. Furthermore, it is also preferred in case of existence of measurement noise in the feedback or simply it is expensive to utilize a sensor.

In this study, a simple Luenberger observer is designed to estimate velocity of the DR. In the real system, the velocity is derived from an encoder output which generated excessive measurement noise. Thus the control signal generated by acceleration feedback has excessive noise which deteriorates the performance of vibration suppression. One may utilize low pass filter to eliminate the noise of the feedback but every filter introduces a phase shift to the signal which is a function of the input signal frequency. This may be a solution for many systems with noise but it is not feasible especially when the control signal is function of time delay. Time delay itself is a pure phase shift and any other phase shift effects the performance of delayed feedback controller. In this sense, utilizing an observer is a feasible solution for this system.

In observer structures observer poles have to be located at a position such that observer states settles at least five times faster than the controllers' to ensure that the error between the states of the observer and the system approaches to zero as soon as possible, as mentioned in Ogata (2010).

The observer is designed using non delayed model of the system and control signal *u(t)* is generated utilizing delayed states. The DR structure with observer is depicted as in Fig. 3.

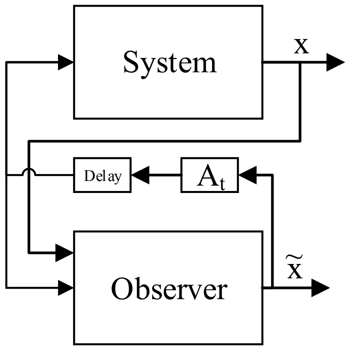

Fig. 2. Observer-Based Delayed Resonator

The proposed method is verified with an experimental study detailed in following section.

3. EXPERIMENT AND RESULTS

Quanser® Active Suspension System and Matlab®/Simulink® environment is used for the implementation of observer-based delayed resonator (Fig. 3). Analogous to Fig. 1 the red plate with the additional mass (B) is considered as the primary structure while the blue plate (A) is utilized as the absorber mass. A DC motor with a capstan mechanism (D) is used to actuate both of the plates which corresponds u(t) in Fig 1. The vibrations are induced to the primary structure by shaking the grey plate (C) at the desired frequency. Physical parameters of the experimental setup are given in Table 1.

Experiments are conducted for the classical delayed resonator both with the sensor feedback of the system and observer-based feedback. The delayed resonator parameter τ and g are obtained using equation (3) and equation (4). Computed controller gains and time delays for targeted undesired vibration frequencies are summarized in Table 2. The excitation frequency ω is set to 3.3, 3.4 and 3.5 rad/sec for all the steps of experiments.

Table 1. The Parameters of the System

Parameter	Value
Mass of Absorber mass (m_a)	2.45 kg
Mass of Primary mass (m_p)	2.807 kg
DR Stiffness (k_a)	900 N/m
Primary Stiffness (k_p)	2500 N/m
DR Damping (b_a)	5.8 N.s/m
Primary Damping (b_p)	5 N.s/m

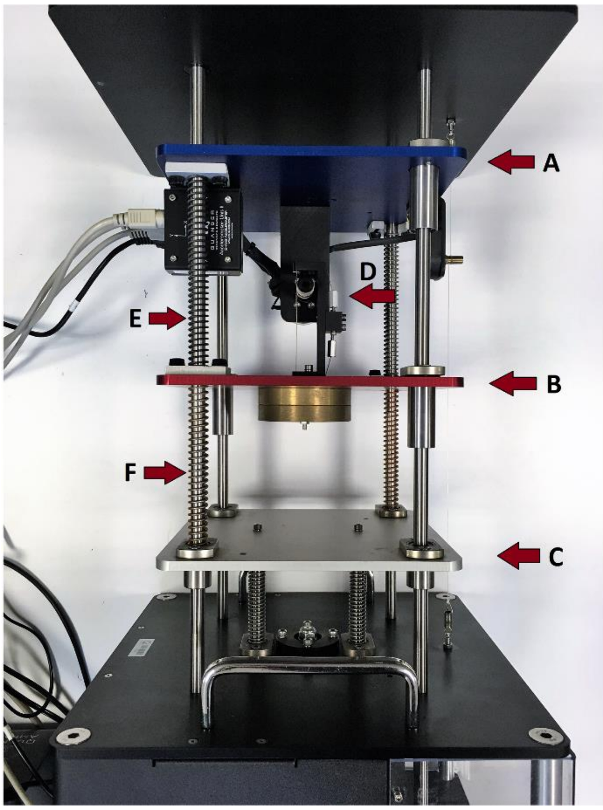

Fig. 3. Experimental setup. (A: Absorber mass, B: Primary mass, C: Shaker, D: Actuator, E: Absorber spring, F: Primary spring)

3.1 DR implementation

First experiment is conducted for excitation frequency of 3.3Hz. DR parameters are computed as g = 0.4532, τ_1 = 0.0321, τ_2 = 0.3351, and τ_3 = 0.6381 using physical parameters of the setup. Here τ_1, belongs to first *g-τ* curves branch and time delay value is quite small for physical system implementation. Thus, in all three experiments, τ_2 values are selected as the feedback delays due to better applicability with the real time system. Absorption performance of the observer-based and classical delayed resonators are compared as shown in Fig. 4a.

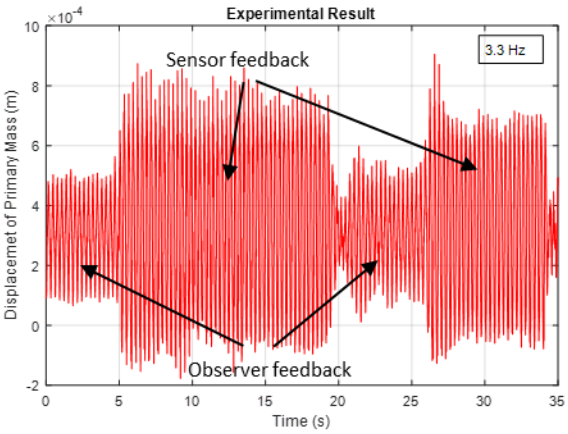

Fig. 4a. Displacement of Primary Mass with Sensor Feedback and Observer Feedback for 3.3Hz

In second and third experiments, excitation frequencies are 3.4 and 3.5 Hz, respectively. The feedback parameters are selected from the Table 2 and results are depicted in Fig.4b and Fig. 4c respectively.

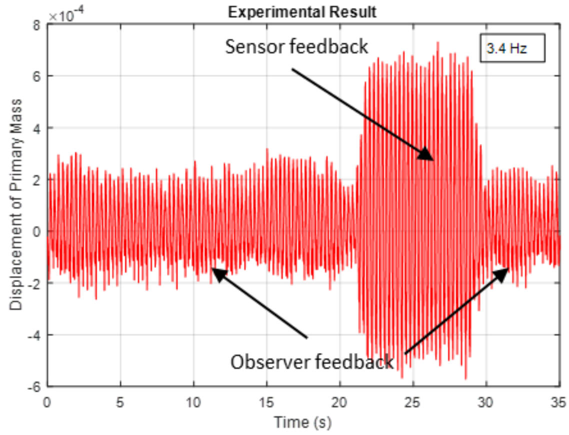

Fig. 4b. Displacement of Primary Mass with Sensor Feedback and Observer Feedback for 3.4Hz

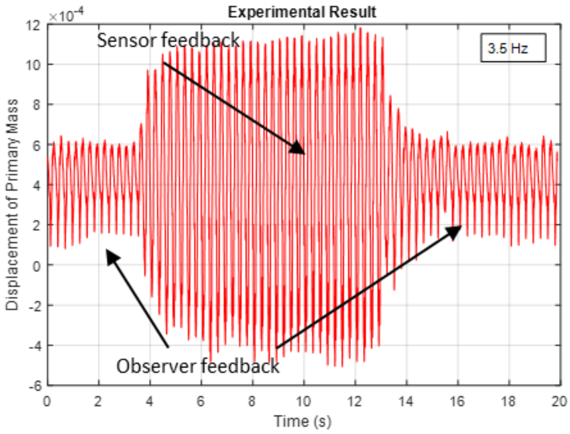

Fig. 4c. Displacement of Primary Mass with Sensor Feedback and Observer Feedback for 3.5Hz

The comparison of vibration suppression performance of DR with sensor feedback and observe feedback, the observer based DR clearly outperforms sensor based DR as expected. As proposed in the article, observer-based structure generates noise free smooth feedback with minimum phase shift which is beneficial for many applications. Naturally, this kind of observer based DRs are more complex thus they need higher computational power which is available now as compared to first implementation period of DRs.

Table 2. Controller Parameters

ω (Hz)	*g* (kg)	τ_1 (s)	τ_2 (s)	τ_3 (s)
3.3	0.4532	0.0321	0.3351	0.6381
3.4	0.5497	0.0242	0.3183	0.6124
3.5	0.6454	0.0191	0.3049	0.5906

3.2 Stability Analysis

As mentioned earlier we utilized the method for determining the imaginary axis eigenvalues of Louisell(2002) and CTCR method of Olgac and Sipahi (2003) for the stability analysis.

The analysis revealed that the non-delayed system is stable and stability switchings occur at time delay values given in Table 4. In the table, ST denotes stable operation region and US unstable. It is obvious that second branch time delay values of DR given in Table 3 provides stable operation in second stability pocket of the combined system.

Table 4. Stability Switchings of the System

g (kg)	τ (s)		τ_1 **(s)**		τ_2 **(s)**		τ_3 **(s)**
0.453	0	**ST**	0.040	**US**	0.156	**ST**	0.424
0.549	0	**ST**	0.031	**US**	0.168	**ST**	0.409
0.645	0	**ST**	0.025	**US**	0.177	**ST**	0.397

4. CONCLUSIONS

In this study, observer-based delayed resonator structure is purposed. Performance of the observer based DR is compared with the sensor based delayed resonator. Both systems employs acceleration feedback and absorber parameters remains the same. The experiments are conducted to validate the better performance of the observer-based delayed resonator. The observer-based delayed resonator achieves faster response and better suppression than classical structure for all three excitation frequencies. Note that an ideal DR would diminish all the vibration but due to uncertainties and the nonlinearities of the system a residual vibration remains. Furthermore, stability analysis revealed stable operation with determined parameters. As the conclusion, it is presented that observer-based resonator may be a successful alternative to classical DR in case of the unavailability of the states or noisy feedback signal.

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