Computing the delay Lyapunov matrix and $\mathcal{H}_2$ norm for high dimensional systems

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The delay Lyapunov matrix plays a fundamental role in constructing Lyapunov functionals of complete type for linear delay systems, in $\mathcal{H}_2$ based model reduction and in $\mathcal{H}_2$ optimal control. Under mild condition it can be found as the unique solution of a boundary value problem with delay, whose dynamics are determined by the the delay Lyapunov equation.

In this talk I will first give an overview of computational methods for solving delay Lyapunov equations and for computing $\mathcal{H}_2$ norm. These are grounded in the collocation and shooting method for solving the boundary value problem (the direct, and semi-analytic approach, respectively), and on a spectral discretization of delay system.

Subsequently I will discuss the computational complexity and convergence properties of these methods, in particular how they scale with the dimension of the original system, and I will explain whether and how they can be adapted to larger-scale systems. It turns out that the results heavily depends on the number of delays and their rational dependency structure, since they affect the smoothness properties of the delay Lyapunov matrix, and the goal of the study (analysis of a given system, or its optimization and control).