# Structural stability in scalar delay equations with monotone feedback

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One of the highest goals in the study of global dynamics is the description of the *global attractor*, a compact structure consisting of all the globally bounded solutions of a system. The possible phenomena happening within the attractor range from the simplest cases e.g. an asymptotically stable equilibrium to more complex settings, which may include chaotic dynamics like in the case of the Lorenz attractor.

Although the analytic description of the global attractor of a system is not possible in general, it is a reasonable challenge in the case of scalar differential delay equations (DDE) with monotone feedback

$$\begin{cases} \dot{x}(t) = f(x(t), x(t-1)), \; f \in C^2(\mathbb{R}^2, \mathbb{R}), \\ \partial_u f(u,v) > 0 \,. \end{cases} \tag{1}$$

For (1) it was proved in [3] that the only possible limit sets for globally bounded solutions are either equilibria or periodic orbits, i.e. solutions of the equation $f(x,x) = 0$ or solutions $x(t)$ of (1) for which there exists some minimal period $T > 0$ such that $x(t+T) = x(t)$. This is a consequence of a *nodal property* measuring the frequency with which two different solutions of (1) cross each other.

This insight on the global dynamics (1) was used in [2] to prove structural stability of the global attractor under a hyperbolicity assumption.

**Theorem 1** *Consider the equation (1) generating an eventually compact, dissipative semi-flow. If the equilibria and periodic solutions of (1) are hyperbolic, then the global attractor is structurally stable.*

The meaning of Theorem 1 is that as long as the equilibria and periodic solutions of (1) have a minimal center dimension, small perturbations of the original nonlinearity $f$ in (1) will leave the global attractor unchanged up to a homeomorphism preserving the time direction of the orbits. This is of particular interest in the understanding of the global effects that local non-invasive control terms may induce. For example if one considers a DDE with a non-invasive Pyragas control term

$$\begin{cases} \dot{x}(t) = g(x(t-1)) + k(x(t) + x(t-\tau)), \; g \in C^2(\mathbb{R}^2, \mathbb{R}), \\ g'(x) > 0 \text{ and } g \text{ odd.} \end{cases} \tag{2}$$

The local behavior of the solutions on (2) near a periodic orbit $x^*(t)$ satisfying the non-invasiveness condition $x^*(t) + x^*(t-\tau) = 0$ was studied in depth in [1]. However, the global consequences of the addition of the control term $k(x^*(t) + x^*(t-\tau))$ still remain unexplored.

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[3] Mallet-Paret J., Sell G., The Poincaré-Bendixson theorem for monotone cyclic feedback systems with delay, *Journal of Dierential Equations* **125**, 1996.