Adaptive Robust Stabilization of Uncertain Neutral Delay–Time Systems via Control Schemes with Simple Structure

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Abstract: In this paper, the problem of robust stabilization is considered for a class of uncertain neutral time–delay dynamical systems with the unknown bounds of delayed state perturbations. By introducing an adaptation law with σ–modification to update the unknown bounds, some continuous adaptive robust state feedback control schemes with a rather simple structure are proposed. The proposed adaptive robust control schemes can guarantee that the solutions of uncertain neutral time–delay systems converge uniformly exponentially towards a ball, and can be easily implemented in practical engineering control problems because of their simplicity. Finally, the simulations of a numerical example are provided to demonstrate the validity of the theoretical results of the paper.

Keywords: Neutral time–delay systems, uncertainties, robust control, adaptive control, state feedback, uniform exponential boundedness.

1. INTRODUCTION

Strictly speaking, some degree of time delays should be involved in almost all of practical engineering systems, such as chemical processes, economic systems, communication networks, biological systems, rolling mill systems, and so on. It is well known that the existence of the delays should lower the performances of engineering control systems, and even results in the instability of such control systems. Thus, the robust stabilization problem of time–delay dynamical systems with delayed state perturbations has attracted much attention, and some approaches have been developed to designing the robust control schemes so that the stability can be guaranteed for time–delay dynamical systems with delayed state perturbations (see, e.g. some standard references: Wu and Mizukami (1996), Oucheriah (2000), Niculescu (2001), Niu et al. (2005), Lin et al. (2005), and the references therein).

On the other hand, there are also some time–delay dynamical systems which are described by neutral functional differential equations. In general, such a class of time–delay dynamical systems is called neutral time-delay systems where the derivatives of the state with time–delay are also included (see, e.g. Kolmanovskii and Nosov (1986), Hale and Lunel (1993)). In the control literature on uncertain delay–time dynamical systems, many adaptive robust control schemes have been proposed for the time–delay systems described retarded functional differential equations (see, e.g. Wu (2000a, 2009, 2012, 2013, 2017), Hua et al. (2008), Zhang et al. (2015), and the references therein). However, there are few results on adaptive robust stabilization of uncertain neutral time–delay dynamical systems since the derivatives of the delayed state appear at the systems (see, e.g. Sun and Zhao (2004), Moezzi and Aghdam (2013), and the references therein). In Sun and Zhao (2004), for instance, a class of adaptive robust state feedback control laws with switching type is proposed for uncertain neutral time–delay dynamical systems, which might result in the chattering of the state responses. In a recent work (Moezzi and Aghdam (2013)), a continuous adaptive robust control scheme is proposed to guarantee some type of stability of uncertain neutral time–delay dynamical systems. However, the control schemes proposed in Moezzi and Aghdam (2013) have a relative complicated structure so that it is difficult to implement such a class of control schemes in practical engineering systems.

In this paper, we also consider the problem of robust stabilization for a class of uncertain neutral time–delay dynamical systems with the unknown bounds of delayed state perturbations. For such a class of uncertain neutral time–delay dynamical systems, we want to develop a class of continuous state feedback control schemes with a rather simple structure. For this purpose, by introducing an adaptation law with σ–modification to update the unknown bounds, we propose some continuous simple adaptive robust state feedback control schemes which can guarantee that the solutions of uncertain neutral time–delay systems converge uniformly exponentially towards a ball, in the presence of delayed state perturbations.

The remainder of the paper consists of the following parts. In Section 2, the problem to be tackled is described. In Section 3, a class of continuous state feedback control schemes with a rather simple structure is constructed. In Section 4, a numerical example is given, and the corresponding simulations are provided to demonstrate the validity of the theoretical results. Finally, the paper concludes in Section 5 with some remarks.

2. PROBLEM FORMULATION

We consider a class of uncertain neutral time–delay dynamical systems described by

$$\dot{x}(t) = Ax(t) + D\dot{x}(t-h_0) + Bu(t) + \sum_{j=1}^{\nu} \Delta G_j(\varsigma,t)x(t-h_j) \tag{1}$$

where $t \in R^+$ is the "time", $x(t) \in R^n$ is the current value of the state, $u(t) \in R^m$ is the control vector, A, B, and D are some constant matrices of appropriate dimensions, and $\Delta G_j(\cdot)$, $j = 1, 2, \ldots, \nu$, are the system uncertainties, which are assumed to be continuous in all their arguments. Here, the uncertain parameter $\varsigma(t)$ is any bounded function in the time. Moreover, for each $j \in \{0, 1, 2, \ldots, \nu\}$, the time delay h_j is assumed to be any nonnegative constant.

The initial condition for system (1) is given by

$$x(t) = \xi(t), \quad t \in [t_0-\bar{h},\ t_0] \tag{2}$$

where $\bar{h} = \max\Big\{h_j,\ j = 0, 1, 2, \ldots, \nu\Big\}$ and $\xi(t)$ is a given continuous function on $[t_0 - \bar{h},\ t_0]$.

Now, the problem is to synthesize a state feedback controller $u(t)$ such that some types of stability of uncertain neutral delay system (1) can be guaranteed in the presence of the uncertainties and delayed state perturbations.

Before proposing our state feedback control scheme, we introduce for (1) the following standard assumptions.

Assumption 2.1. For the uncertain matrices, there exist some continuous and bounded matrix functions $H_j(\cdot)$ of appropriate dimensions such that

$$\Delta G_j(\varsigma,t) = BH_j(\varsigma,t), \quad j = 1, 2, \ldots, \nu$$

In this paper, we want to synthesize some types of stabilizing state feedback controllers of uncertain neutral time–delay systems with unknown upper bounds of uncertainties. More concretely speaking, the unknown upper bounds of the uncertainties are described in the following form.

$$\rho_j(t) := \max_{\varsigma} \lambda_{\max}\left(H_j(\varsigma,t)H_j^\top(\varsigma,t)\right), \quad j = 1, 2, \ldots, \nu$$

where $\lambda_{\max}(\cdot)$ stands for the maximum eigenvalue of the matrix (see, e.g. Wu (2000b)). Here, for each $j \in \{0, 1, 2, \ldots, \nu\}$, the function $\rho_j(t)$ is assumed to be completely unknown. Moreover, the uncertain $\rho_j(t)$ is also assumed, without loss of generality, to be uniformly continuous and bounded for any $t \in R^+$.

Assumption 2.2. There exist some symmetric positive definite matrices $P \in R^{n\times n}$, $R \in R^{n\times n}$, $S \in R^{n\times n}$, and some positive constants η and κ such that the following conditions can be satisfied.

$$PA + A^\top P + PADR^{-1}D^\top A^\top P + S + S^\top - \eta PBB^\top P \ < \ 0 \tag{3a}$$

$$2D^\top SD - \frac{1}{2}\mathrm{e}^{-\kappa h_0}S + R \ \leq \ 0 \tag{3b}$$

In addition, we also introduce the following standard assumptions in the control literature on robust stabilization of uncertain neutral time–delay systems (see, e.g. Kharitonov et al. (2005) and the references therein).

Assumption 2.3. It is assumed that the norm of the constant matrix D is less than one; i.e. $\|D\| < 1$.

Moreover, we also introduce a stability definition.

Definition 2.1. (*Exponential boundedness*) Consider the retarded functional differential equation

$$\frac{dx(t)}{dt} \ = \ f(t, x_t) \tag{4}$$

with the initial condition

$$x(t) \ = \ \chi(t), \quad t \in [t_0 - h,\ t_0]$$

where $x_t \ := \ x(t+\theta),\ \theta \in [-h,\ 0]$.

Then, the dynamical systems described by (4) are said to be exponentially bounded, if there exist some positive constants ε, α, and $\kappa(\delta) > 0$ such that for any $\delta > 0$ and for any $t > t_0$,

$$\big\|x(t)\big\| \ \leq \ \kappa(\delta)\mathrm{e}^{\alpha(t-t_0)} + \varepsilon \tag{5}$$

where $\displaystyle\sup_{\tau\in[t_0-h,\ t_0]} \big\|x(\tau)\big\| \ < \ \delta$.

Moreover, if for any $t_0 \in R^+$, inequality (5) holds, then the dynamical systems described by (4) are also said to be uniformly exponentially bounded.

3. SIMPLE CONTROL SCHEMES

Before giving the main result of this paper, we first introduce some definitions on the unknown upper bounds of the uncertainties and delayed state perturbations.

In this paper, since the upper bound $\rho_j(t)$ has been assumed to be continuous and bounded for any $t \in R^+$, it is obvious that there exist some positive constants ρ_j^*, $j = 1, 2, \ldots, \nu$, which are defined by

$$\rho_j^* := \max\left\{\ \rho_j(t)\ :\ t \in R^+\ \right\} \tag{6}$$

Here, it is also obvious that the constants ρ_j^*, $j = 1, 2, \ldots, \nu$, are still unknown. Therefore, such unknown upper bounds can not be directly employed to construct a state feedback controller.

For our synthesis method, we also introduce a definition on the unknown upper bounds in the form of

$$\psi^* \;:=\; \varrho^{-1}\Big\{\eta + 2\nu\sum_{j=1}^{\nu} \mathrm{e}^{\kappa h_j}\rho_j^* \lambda_{\min}^{-1}(S)\Big\} \tag{7}$$

where ϱ, η, and κ are any given positive constants. It is obvious from (7) that ψ^* is still an unknown positive constant.

Moreover, for simplicity, we also define

$$\Pi_{h_0}(x(t)) \;=\; x(t) - Dx(t-h_0) \tag{8}$$

Now, for the uncertain neutral time–delay systems described by (1), we proposed the robust state feedback control schemes in the form of

$$\begin{aligned} u(t) =&\; p(x(t),\hat{\psi}(t),t) \\ =&-\frac{1}{2}\,\varrho\,\hat{\psi}(t)B^\top P\,\Pi_{h_0}(x(t)) \end{aligned} \tag{9}$$

where $\hat{\psi}(t)$ is the estimate of the unknown ψ^* which is updated by the following adaptive law:

$$\frac{d\hat{\psi}(t)}{dt} \;=\; -\gamma\sigma\hat{\psi}(t) + \gamma\varrho\left\|B^\top P\,\Pi_{h_0}(x(t))\right\|^2 \tag{10}$$

where γ and σ are any given positive constants.

By applying (9) to (1), we can obtain an uncertain closed–loop neutral time–delay system of the form

$$\begin{aligned} \dot{x}(t) =&\; Ax(t) - \frac{1}{2}\,\varrho\,\hat{\psi}(t)BB^\top P\,\Pi_{h_0}(x(t)) \\ &+D\dot{x}(t-h_0) + \sum_{j=1}^{\nu}\Delta G_j(\varsigma,t)x(t-h_j) \end{aligned} \tag{11}$$

On the other hand, by defining $\tilde{\psi}(t) := \hat{\psi}(t) - \psi^*$, from (10) we can also have the error systems on adapation law, described by the following equation.

$$\frac{d\tilde{\psi}(t)}{dt} = -\gamma\sigma\tilde{\psi}(t) + \gamma\varrho\left\|B^\top P\,\Pi_{h_0}(x(t))\right\|^2 - \gamma\sigma\psi^* \tag{12}$$

In the following, by $(x,\tilde{\psi})(t)$ we denote a solution of the closed–loop neutral time–delay systems and the error systems described by (11) and (12). Then, the following theorem can be obtained which shows the solutions of the closed–loop neutral time–delay systems and the error systems are uniformly exponentially bounded.

Theorem 3.1. Consider the adaptive closed–loop neutral time–delay dynamical systems described by (11) and (12). Suppose that *Assumptions 2.1* to *2.3* are satisfied. Then, the solutions $(x,\tilde{\psi})\ (t;\ t_0, x(t_0), \tilde{\psi}(t_0))$ of the closed–loop neutral time–delay neutral systems described by (11) and the error systems described by (12) are uniformly exponentially bounded in the presence of the uncertainties and delayed state perturbations.

Proof: For the neutral time–delay dynamical systems described by (11) and (12), we construct a Lyapunov–Krasovskii functional candidate of the form:

$$\begin{aligned} V(x,\tilde{\psi}) =&\; \Pi_{h_0}^\top(x(t))P\,\Pi_{h_0}(x(t)) + \frac{1}{2}\gamma^{-1}\tilde{\psi}^2(t) \\ &+\frac{1}{2}\int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds \\ &+\frac{1}{2}\sum_{j=1}^{\nu}\nu^{-1}\int_{t-h_j}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds \end{aligned} \tag{13}$$

where $P \in R^{n\times n}$ and $S \in R^{n\times n}$, are some positive definite matrices, and κ is any given positive constant.

Let $(x(t),\ \tilde{\psi}(t))$ be the solution of (11) and (12) for $t \geq t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (11) and (12) we can obtain that for $t \geq t_0$,

$$\begin{aligned} \frac{dV(x,\tilde{\psi})}{dt} =&\; \Pi_{h_0}^\top(x(t))\Big(PA + A^\top P\Big)\Pi_{h_0}(x(t)) \\ &+2\Pi_{h_0}^\top(x(t))PADx(t-h_0) \\ &-\varrho\,\hat{\psi}(t)\Pi_{h_0}^\top(x(t))PBB^\top P\,\Pi_{h_0}(x(t)) \\ &+2\Pi_{h_0}^\top(x(t))P\sum_{j=1}^{\nu}\Delta G_j(\varsigma,t)x(t-h_j) \\ &+\frac{1}{2}x^\top(t)Sx(t) - \frac{1}{2}\mathrm{e}^{-\kappa h_0}x^\top(t-h_0)Sx(t-h_0) \\ &-\frac{1}{2}\kappa\int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \\ &+\frac{1}{2}\sum_{j=1}^{\nu}\nu^{-1}x^\top(t)Sx(t) \\ &-\frac{1}{2}\sum_{j=1}^{\nu}\nu^{-1}\mathrm{e}^{-\kappa h_j}x^\top(t-h_j)Sx(t-h_j) \\ &-\frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds \\ &+\gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \end{aligned} \tag{14}$$

Notice the fact that for any positive definite matrix $M > 0$,

$$2X^\top Y \;\leq\; X^\top MX + Y^\top M^{-1}Y, \quad \forall\, X,Y \in R^l$$

Then, we can obtain two inequalities in the form of

$$\begin{aligned} 2\Pi_{h_0}^\top(x(t))PADx(t-h_0) \;\leq&\; x^\top(t-h_0)Rx(t-h_0) \\ &+\Pi_{h_0}^\top(x(t))PADR^{-1}D^\top A^\top P\,\Pi_{h_0}(x(t)) \end{aligned} \tag{15}$$

and

$$2\Pi_{h_0}^\top(x(t))P\sum_{j=1}^{\nu}\Delta G_j(\varsigma,t)x(t-h_j)$$

$$\leq \sum_{j=1}^{\nu} 2\nu \mathrm{e}^{\kappa h_j} \Pi_{h_0}^\top(x(t)) P \Delta G_j(\varsigma,t) S^{-1} \times \Big(\Delta G_j(\varsigma,t)\Big)^\top P\,\Pi_{h_0}(x(t)) + \frac{1}{2}\sum_{j=1}^{\nu} \nu^{-1}\mathrm{e}^{-\kappa h_j} x^\top(t-h_j) S x(t-h_j) \quad (16)$$

Therefore, substituting (15) and (16) into (14) yields that for $t \geq t_0$,

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} \leq\, & \Pi_{h_0}^\top(x(t))\Big(PA + A^\top P\Big)\Pi_{h_0}(x(t)) \\
& + \Pi_{h_0}^\top(x(t)) PADR^{-1}D^\top A^\top P\,\Pi_{h_0}(x(t)) \\
& - \varrho\,\hat{\psi}(t)\Pi_{h_0}^\top(x(t)) PBB^\top P\,\Pi_{h_0}(x(t)) \\
& + \sum_{j=1}^{\nu} 2\nu \mathrm{e}^{\kappa h_j}\Pi_{h_0}^\top(x(t)) P\Delta G_j(\varsigma,t)S^{-1} \times \Big(\Delta G_j(\varsigma,t)\Big)^\top P\,\Pi_{h_0}(x(t)) \\
& + \frac{1}{2}x^\top(t)Sx(t) - \frac{1}{2}\mathrm{e}^{-\kappa h_0}x^\top(t-h_0)Sx(t-h_0) \\
& - \frac{1}{2}\kappa \int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\, x^\top(s)Sx(s)ds \\
& + \frac{1}{2}\sum_{j=1}^{\nu}\nu^{-1}x^\top(t)Sx(t) + x^\top(t-h_0)Rx(t-h_0) \\
& - \frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t}\mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \\
& + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \\
=\, & \Pi_{h_0}^\top(x(t))\Big(\, PA + A^\top P + PADR^{-1}D^\top A^\top P\,\Big)\Pi_{h_0}(x(t)) \\
& + \sum_{j=1}^{\nu} 2\nu \mathrm{e}^{\kappa h_j}\Pi_{h_0}^\top(x(t)) P\Delta G_j(\varsigma,t)S^{-1} \times \Big(\Delta G_j(\varsigma,t)\Big)^\top P\,\Pi_{h_0}(x(t)) \\
& + x^\top(t-h_0)Rx(t-h_0) \\
& + x^\top(t)Sx(t) - \frac{1}{2}\mathrm{e}^{-\kappa h_0}x^\top(t-h_0)Sx(t-h_0) \\
& - \varrho\,\hat{\psi}(t)\Pi_{h_0}^\top(x(t)) PBB^\top P\,\Pi_{h_0}(x(t)) \\
& - \frac{1}{2}\kappa \int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\, x^\top(s)Sx(s)ds \\
& - \frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t}\mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \\
& + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \qquad (17)
\end{aligned}$$

It can be observed that

$$\begin{aligned}
x^\top(t)Sx(t) =\, & 2\Pi_{h_0}^\top(x(t))SDx(t-h_0) \\
& - 2\Pi_{h_0}^\top(x(t))SDx(t-h_0) + x^\top(t)Sx(t) \\
\leq\, & \Pi_{h_0}^\top(x(t))\Big(\, S + S^\top\,\Big)\Pi_{h_0}(x(t)) \\
& + 2x^\top(t-h_0)D^\top SDx(t-h_0) \qquad (18)
\end{aligned}$$

Thus, applying (18) into (17) yields that for $t \geq t_0$,

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} \leq\, & \Pi_{h_0}^\top(x(t))\Big(\, PA + A^\top P + S + S^\top + PADR^{-1}D^\top A^\top P\,\Big)\Pi_{h_0}(x(t)) \\
& + x^\top(t-h_0)\Big(2D^\top SD - \frac{1}{2}\mathrm{e}^{-\kappa h_0}S + R\Big)x(t-h_0) \\
& + \sum_{j=1}^{\nu} 2\nu \mathrm{e}^{\kappa h_j}\Pi_{h_0}^\top(x(t)) P\Delta G_j(\varsigma,t)S^{-1} \times \Big(\Delta G_j(\varsigma,t)\Big)^\top P\,\Pi_{h_0}(x(t)) \\
& - \varrho\,\hat{\psi}(t)\Pi_{h_0}^\top(x(t)) PBB^\top P\,\Pi_{h_0}(x(t)) \\
& - \frac{1}{2}\kappa \int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\, x^\top(s)Sx(s)ds + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \\
& - \frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t}\mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \qquad (19)
\end{aligned}$$

On the other hand, from Assumption 2.1, it can be obtained that

$$\begin{aligned}
& \Pi_{h_0}^\top(x(t))P\Delta G_j(\varsigma,t)S^{-1}\Big(\Delta G_j(\varsigma,t)\Big)^\top P\,\Pi_{h_0}(x(t)) \\
& \quad \leq\ \rho_j^*\lambda_{\min}^{-1}(S)\Pi_{h_0}^\top(x(t))PBB^\top P\Pi_{h_0}(x(t)) \qquad (20)
\end{aligned}$$

Thus, by substituting (20) into (19) we can obtain that

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} \leq\, & -\Pi_{h_0}^\top(x(t))\tilde{Q}\,\Pi_{h_0}(x(t)) \\
& + x^\top(t-h_0)\Big(2D^\top SD - \frac{1}{2}\mathrm{e}^{-\kappa h_0}S + R\Big)x(t-h_0) \\
& + \eta\Pi_{h_0}^\top(x(t))PBB^\top P\Pi_{h_0}(x(t)) \\
& + 2\nu\sum_{j=1}^{\nu}\mathrm{e}^{\kappa h_j}\rho_j^*\lambda_{\min}^{-1}(S)\Pi_{h_0}^\top(x(t))PBB^\top P\Pi_{h_0}(x(t)) \\
& - \varrho\hat{\psi}(t)\Pi_{h_0}^\top(x(t))PBB^\top P\Pi_{h_0}(x(t)) \\
& - \frac{1}{2}\kappa \int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\, x^\top(s)Sx(s)ds + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \\
& - \frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t}\mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \qquad (21)
\end{aligned}$$

where

$$-\tilde{Q} := PA + A^\top P$$
$$+PADR^{-1}D^\top A^\top P + S + S^\top - \eta PBB^\top P$$

It is obvious from Assumption 2.2 that $\tilde{Q}$ is a positive definite matrix. Therefore, from (7), (21) and Assumption 2.2 we can obtain that for $t \geq t_0$,

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} &\leq -\lambda_{\min}\left(\tilde{Q}\right)\left\|\Pi_{h_0}(x(t))\right\|^2 \\
&\quad +\left(\eta + 2\nu\sum_{j=1}^{\nu} \mathrm{e}^{\kappa h_j}\rho_j^*\lambda_{\min}^{-1}(S)\right)\left\|B^\top P\,\Pi_{h_0}(x(t))\right\|^2 \\
&\quad -\varrho\hat{\psi}(t)\Pi_{h_0}^\top(x(t))PBB^\top P\Pi_{h_0}(x(t)) \\
&\quad -\frac{1}{2}\kappa\int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \\
&\quad -\frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t} \mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \\
&= -\lambda_{\min}\left(\tilde{Q}\right)\left\|\Pi_{h_0}(x(t))\right\|^2 \\
&\quad +\varrho\psi^*\left\|B^\top P\,\Pi_{h_0}(x(t))\right\|^2 - \varrho\hat{\psi}(t)\left\|B^\top P\,\Pi_{h_0}(x(t))\right\|^2 \\
&\quad -\frac{1}{2}\kappa\int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds + \gamma^{-1}\tilde{\psi}(t)\frac{d\tilde{\psi}(t)}{dt} \\
&\quad -\frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t} \mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds \qquad (22)
\end{aligned}$$

Then, by introducing (12) into (22) we can obtain the following inequality:

$$\begin{aligned}
\frac{dV(x,\tilde{\psi})}{dt} &\leq -\lambda_{\min}\left(\tilde{Q}\right)\lambda_{\max}^{-1}\left(P\right)\Pi_{h_0}^\top(x(t))P\,\Pi_{h_0}(x(t)) \\
&\quad -\frac{1}{2}\sigma\tilde{\psi}^2(t) - \frac{1}{2}\kappa\int_{t-h_0}^{t} \mathrm{e}^{\kappa(s-t)}\,x^\top(s)Sx(s)ds \\
&\quad -\frac{1}{2}\sum_{j=1}^{\nu}\kappa\nu^{-1}\int_{t-h_j}^{t} \mathrm{e}^{\kappa(s-t)}x^\top(s)Sx(s)ds + \frac{1}{2}\sigma\left|\psi^*\right|^2 \qquad (23)
\end{aligned}$$

Thus, from (23) we can further obtain that for $t \geq t_0$,

$$\frac{dV(x,\tilde{\psi})}{dt} \;\leq\; -\alpha V(x,\tilde{\psi}) + \tilde{\varepsilon} \qquad (24)$$

where

$$\alpha := \min\left\{\lambda_{\min}\left(\tilde{Q}\right)\lambda_{\max}^{-1}\left(P\right), \quad \sigma\gamma, \quad \kappa\right\}$$
$$\tilde{\varepsilon} := \frac{1}{2}\sigma\left|\psi^*\right|^2$$

For the sake of convenience, we define

$$V(t) \;:=\; V(x(t),\tilde{\psi}(t)))$$

Thus, from (24) we can obtain that for any $t \geq t_0$,

$$\frac{dV(t)}{dt} \;\leq\; -\alpha V(t) + \tilde{\varepsilon} \qquad (25)$$

which results in an inequality on $V(t)$ as follows. That is, for any $t \geq t_0$,

$$V(t) \leq \exp\{-\alpha(t-t_0)\}V(t_0) + \tilde{\varepsilon}\alpha^{-1} \qquad (26)$$

It is obvious from (13) and (26) that for any $t \geq t_0$,

$$\begin{aligned}
\lim_{t\to\infty}\left\|\Pi_{h_0}(x(t))\right\| \;&=\; \lim_{t\to\infty}\left\|x(t) - Dx(t-h_0)\right\| \\
&\leq\; \sqrt{\tilde{\varepsilon}\alpha^{-1}\lambda_{\min}^{-1}(P)} \;:=\; \theta^* \qquad (27)
\end{aligned}$$

In the light of Assumption 2.3, by employing the similar method which has been used in Moezzi and Aghdam (2013) and Kharitonov et al. (2005), we can obtain that for any $t \geq t_0$,

$$\left\|x(t)\right\| \;\leq\; \theta^*\left(1 + \frac{\tilde{\varepsilon}}{1-\|D\|}\right) + \|\xi(0)\|\chi\mathrm{e}^{-\gamma^* t} \qquad (28)$$

where χ and γ^* are two positive constants. It follows from (13), (26) and (28) that $(x,\tilde{\psi})$ $(t;\, t_0, x(t_0), \tilde{\psi}(t_0))$ of the closed–loop neutral time–delay neutral systems described by (11) and the error systems described by (12) are uniformly exponentially bounded in the presence of the uncertainties, external disturbance, and delayed state perturbations. Thus, we complete this proof. $\nabla\nabla\nabla$

4. ILLUSTRATIVE EXAMPLE

In this section, similar to Moezzi and Aghdam (2013) we also consider the following numerical example.

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \dot{x}(t-h_0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \\
&\quad + \sum_{j=1}^{3} \Delta G_j(\varsigma,t)x(t-h_j) \qquad (29)
\end{aligned}$$

where the neutral delay h_0 is set as $h_0 = 0.5$, and for simulation, the uncertain $\Delta G_j(\varsigma,t)$, $j = 1,2,3$, are given as follows.

$$\Delta G_1(\varsigma,t) = \begin{bmatrix} 0 & 0 \\ \varsigma(t) & \varsigma(t) \end{bmatrix}, \quad \Delta G_2(\varsigma,t) = \begin{bmatrix} 0 & 0 \\ 0 & \varsigma(t) \end{bmatrix}$$

$$\Delta G_3(\varsigma,t) = \begin{bmatrix} 0 & 0 \\ 1.5\varsigma(t) & \varsigma(t) \end{bmatrix}$$

Thus, similar to Moezzi and Aghdam (2013), in the light of *Assumption 2.2*, the following parameters can be obtained.

$$P = \begin{bmatrix} 0.0154 & 0.0279 \\ 0.0279 & 0.0581 \end{bmatrix}, \quad \eta = 250.56, \quad \kappa = 1.0$$

That is, such parameters will guarantee that the condition described by (3) can be satisfied.

For the adaptation law described by (10), we choose

$$\gamma = 0.5, \quad \sigma = 0.15, \quad \varrho = 20$$

Thus, the adaptive robust controllers described by (9) and (10) can guarantee the uniform ultimate boundedness of uncertain neutral time–delay systems.

For simulation, the uncertain time–varying parameter $\varsigma(t)$, the constant time delays h_j, and initial conditions are given as follows.

$$\begin{aligned} &h_0 = 0.5, \quad h_1 = 1.0, \quad h_2 = 2.0, \quad h_3 = 3.0 \\ &x(0) = [\, 8.0\cos(t) \ \ -8.0\cos(t)\,]^{\top}, \quad t \in [-\bar{h},\ 0] \\ &\hat{\psi}(0) = 8.0, \quad \varsigma(t) = 0.1\sin(2t) \end{aligned}$$

The simulation results have been shown in *Figs.1–2* for this numerical example. The state responses of system (29) have been depicted in *Fig.1*, which shows the stability of the closed–loop neutral time–delay systems under the simple adaptive robust control schemes described by (9) and (10). In addition, the control input has been shown in *Fig.2*, which is also convergent toward zero as the state variables decrease. From *Fig.1* and *Fig.2*, we can observe that the proposed adaptive robust control schemes with a rather simple structure indeed make it stable for the uncertain neutral time–delay systems.

(The details of the illustrative numerical example and the figures of the simulation will be displayed in the oral presentation.)

5. CONCLUDING REMARKS

In this paper, the problem of robust stabilization has been considered for a class of uncertain neutral time–delay dynamical systems with the unknown bounds of delayed state perturbations. Some continuous adaptive robust state feedback control schemes with a rather simple structure have been proposed. It has shown that the proposed adaptive robust control schemes can guarantee that the solutions of uncertain neutral time–delay systems converge uniformly exponentially towards a ball, and can be easily implemented in practical engineering control problems because of their simplicity. Moreover, the method employed in this paper can be applied to a rather large class of uncertain neutral time–delay dynamical systems, and be expected to obtain some interesting results.

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