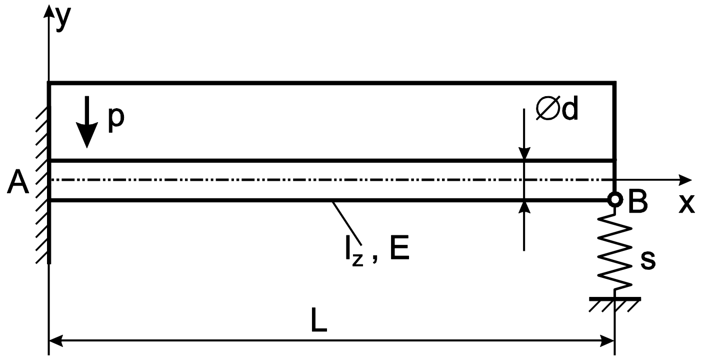
Matrix formulation of Ritz's method, beams subjected to bending

PROBLEM 3.1



In-plane beam with spring support and line load.

The linear elastic built-in beam with circular cross section is supported by a spring and loaded by p. We apply the approximate method of Ritz to calculate the displacement function.

a. Calculate the deflection and angle of rotation function of the beam using MATHEMATICA. Apply 0^{th} – 6^{th} approximations. Record the displacement at cross section B and plot the $v_B - N_{appr}$ relationship.
b. Plot the deformed shape using the 6^{th} approximation function. Calculate and plot the beam diagrams (shear force and bending moment) using the 6^{th} approximation function.

Data: $L = 1.2$ m $d = 46$ mm $E = 200$ GPa $s = 2\cdot10^5$ N/m $p = 750$ N/m

The potential energy of the system is:

$$U < v >= \frac{1}{2}\int_0^l I_z E v''^2\, dx + \frac{1}{2} s v^2(L) - \int_0^l (-p) v(x) dx\,,$$

the kinematic B.C.s are (built-in end):

$$v(0) = 0\,,\; v'(0) = 0\,.$$

the simplest function satisfying these conditions is (ω(x)):

$$\omega(x) = x^2\,,$$

and the displacement function of the beam is:

$$v(x) = x^2(a_0 + a_1 x + a_2 x^2 + ...\,) = \underline{B}^T\, \underline{A}\,.$$

MATHEMATICA solution

Definition of the geometrical and material properties (units are in SI)

```
p=750;L=1.2;d=0.046;Iz=d^4*Pi/64;Ex=200*10^9;s=2*10^5;
omega=x^2;
```

Definition of the number of approximation

```
Nappr=0;
```

Definition of the dimension of vector B (vector of basis polynomials)

```
B = omega*Table[{x^(i-1)},{i,1,Nappr+1}];
```

Calculation of B^T, the 1st and 2nd derivative of B

```
Bt=Transpose[B];
dB=D[B,x];
ddB=D[B,{x,2}];
ddBt=Transpose[ddB];
```

Construction of the stiffness matrix and the vector of external forces

```
S=Integrate[Iz*Ex*ddB.ddBt,{x,0,L}]+s*(B.Bt)/.x->L;
Q=Integrate[-p*B,{x,0,L}]
```

Calculation of the vector of unknown coefficients

```
A=Inverse[S].Q;
```

Deflection and angle of rotation function

```
v[x_]=Transpose[B].A;
phi[x_]=D[v[x],x];
```

Calculate the deflection and the rotation at cross section B

```
vb=v[L]
phib=phi[L]
```

Convergence

```
convvb={};
convphib= {};
```

Repeat this procedure until the 6th approximation, run the worksheet again, increase "Nappr" at each step, put the results into the table.

Plot the data

```
ListPlot[convvb]
ListPlot[convphib]
```

Animation of deflection and angle of rotation functions

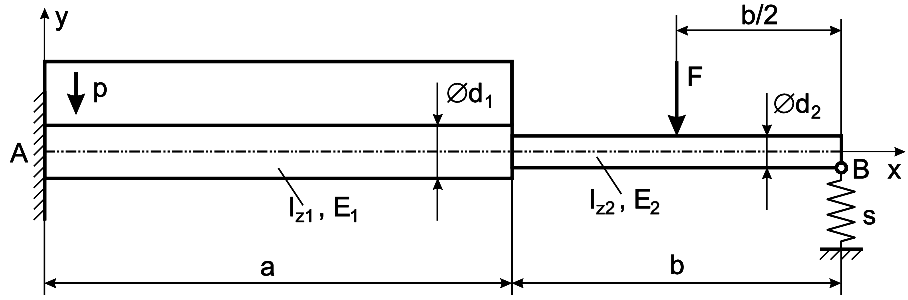
```
Animate[Plot[t*v[x],{x,0,L},PlotRange->{All,{0,-0.001222}},AspectRatio->0.2],{t,0,1}]
Animate[Plot[t*phi[x],{x,0,L},PlotRange->{All,{0,-0.0015}},AspectRatio->0.2],{t,0,1}]
```

Beam diagrams , bending moment and shear force, animation

```
Mb[x_]=Iz*Ex*D[v[x],{x,2}];
V[x_]=-D[Mb[x],x]
Animate[Plot[t*Mb[x],{x,0,L},PlotRange->{All,{-250,40}},AspectRatio->0.2],{t,0,1}]
Animate[Plot[t*V[x],{x,0,L},PlotRange->{All,{-670,250}},AspectRatio->0.2],{t,0,1}]
```

Results: (6th approximation): V_A = 655.8 N, M_A = 246.9 Nm, V_B = -244.3 N, M_B = 0 Nm,

PROBLEM 3.2



In-plane beam with piecewise continuous cross section.

a. *Calculate the deflection and angle of rotation function of the beam. Apply 0th – 8th approximations. Record the displacement at cross section B and plot the $v_B - N_{appr}$ relationship.*
b. *Plot the deformed shape using the 8th approximation function. Calculate and plot the beam diagrams (shear force and bending moment) using the 8th approximation function.*

Data:	$a = 1$ m	$b = 0.8$ m	$d_1 = 60$ mm	$d_2 = 46$ mm
	$E_1 = 210$ GPa	$E_2 = 190$ GPa	$s = 2\cdot10^5$ N/m	$p = 800$ N/m
	$F = 1000$ N			

The potential energy of the system is:

$$U<v>=\frac{1}{2}\int_0^a I_{z1}E_1 v''^2\,dx+\frac{1}{2}\int_a^{a+b} I_{z2}E_2 v''^2\,dx+\frac{1}{2}sv^2(L)-\int_0^a(-p)v(x)dx-(-F)v(a+b/2)$$

Hint: Modify the worksheet of Problem 3.1.....Results: v_B = -2.733 mm, φ_B = -0.000169 rad