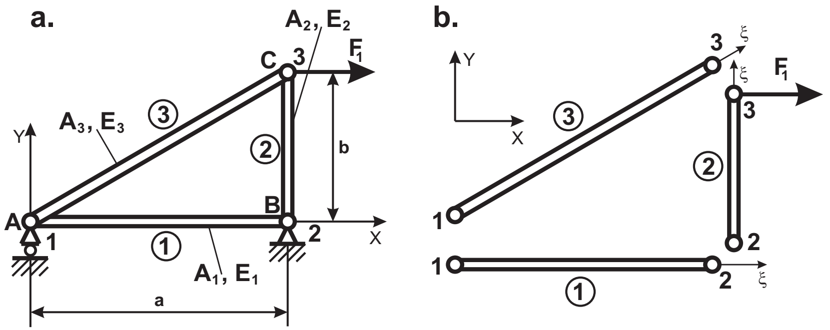
Static analysis of a 2D TRUSS structure

PROBLEM 5.1



2D TRUSS structure.

The beam structure above consists of linear elastic TRUSS elements. Calculate the nodal displacements and the reactions by the finite element method using MATHEMATICA. Animate the deformed shape of the structure. Calculate and display the beam diagram (normal force) of the elements.

Data:	a = 0.6 m	b = 0.4 m	F_1 = 100 kN
	$A_1 = 100 \cdot 10^{-6}$ m^2	$A_2 = 120 \cdot 10^{-6}$ m^2	$A_3 = 90 \cdot 10^{-6}$ m^2
	E_1 = 210 GPa	E_2 = 180 GPa	E_3 = 195 GPa

MATHEMATICA solution

Creation of the element stiffness matrices, local system *(use copy & paste)*

```
Ke1= A1*E1/L1*{{1,-1},{-1,1}};
Ke2= A2*E2/L2*{{1,-1},{-1,1}};
Ke3= A3*E3/L3*{{1,-1},{-1,1}};
```

Definition of the transformation matrices and their transposed matrices *(use copy & paste)*

```
T1={{c1,0},{s1,0},{0,c1},{0,s1}};
T1t=Transpose[T1];
T2={{c2,0},{s2,0},{0,c2},{0,s2}};
T2t=Transpose[T2];
T3={{c3,0},{s3,0},{0,c3},{0,s3}};
T3t=Transpose[T3];
```

Check the orthogonality of the transformation matrices

```
T1t.T1;T2t.T2;T3t.T3;
```

Transformation of stiffness matrices into the global system *(use copy & paste)*

```
Ke1g= T1.Ke1.T1t;
Ke2g= T2.Ke2.T2t;
Ke3g= T3.Ke3.T3t;
```


Definition of matrices containing zeros *(use copy & paste)*

```
K1=Table[0,{6},{6}];
K2=Table[0,{6},{6}];
K3=Table[0,{6},{6}];
```

Expansion of the stiffness matrices in accordance with the DOFs

The structural stiffness matrix is built up in accordance with the figure below (refer to the DOFs, the superscript refers to the element number):

$$\underline{\underline{K}} = \begin{bmatrix} k^1_{11}+k^3_{11} & k^1_{12}+k^3_{12} & k^1_{13} & k^1_{14} & k^3_{13} & k^3_{14} \\ k^1_{21}+k^3_{21} & k^1_{22}+k^3_{22} & k^1_{23} & k^1_{24} & k^3_{23} & k^3_{24} \\ k^1_{31} & k^1_{32} & k^1_{33}+k^2_{11} & k^1_{34}+k^3_{12} & k^2_{13} & k^2_{14} \\ k^1_{41} & k^1_{42} & k^1_{43}+k^3_{21} & k^1_{44}+k^3_{23} & k^2_{23} & k^2_{24} \\ k^3_{31} & k^3_{32} & k^2_{31} & k^2_{32} & k^1_{33}+k^3_{33} & k^1_{34}+k^3_{34} \\ k^3_{41} & k^3_{42} & k^2_{41} & k^2_{42} & k^1_{43}+k^3_{43} & k^1_{44}+k^3_{44} \end{bmatrix}$$

$\underline{\underline{K}}_1$ ——— $\underline{\underline{K}}_2$ – – $\underline{\underline{K}}_3$ ······

```
K1[[{1,2,3,4},{1,2,3,4}]]= K1[[{1,2,3,4},{1,2,3,4}]]+Ke1g;
K2[[{3,4,5,6},{3,4,5,6}]]= K2[[{3,4,5,6},{3,4,5,6}]]+Ke2g;
K3[[{1,2,5,6},{1,2,5,6}]]= K3[[{1,2,5,6},{1,2,5,6}]]+Ke3g;
```

Creation of the structural stiffness matrix

```
K=K1+K2+K3;
```

Definition of B.C.s and the structural nodal displacement vector

```
v1=0;u2=0;v2=0;
U={u1,v1,u2,v2,u3,v3};
```

Definition of the structural load vector

```
F={0,Ay,Bx,By,F1,0};
```

Definition of the geometrical and material properties (units are in SI)

```
a=0.6;b=0.4;A1=100*10^-6;E1=210*10^9;A2=120*10^-6;E2=180*10^9;A3=90*10^-6;
E3=195*10^9;F1=100*10^3;
```

Nodal coordinates and beam element lengths *(use copy & paste)*

```
x1=0;y1=0;x2=a;y2=0;x3=a;y3=b;
L1=Sqrt[(x2-x1)^2+(y2-y1)^2];
L2=Sqrt[(x3-x2)^2+(y3-y2)^2];
L3=Sqrt[(x3-x1)^2+(y3-y1)^2];
```

Display the structure by polygonplot

```
L={{x1,y1},{x2,y2},{x3,y3}};
Poly=Graphics[{{White,EdgeForm[Thick],Polygon[L]}}];
```

Definition of the sine and cosine of the angles

```
s1=(y2-y1)/L1;s2=(y3-y2)/L2;s3=(y3-y1)/L3;
```

```
c1=(x2-x1)/L1;c2=(x3-x2)/L2;c3=(x3-x1)/L3;
```

The structural FE equilibrium equation

```
actdof={1,5,6};
fixdof={2,3,4};
Kc=K[[actdof,actdof]];
Fc=F[[actdof]];
U[[actdof]]=LinearSolve[Kc,Fc]
```

Convert the left and right-hand sides into list

```
displacement=U;
forces=K.displacement;

u1=displacement[[1]];
u3=displacement[[5]];
v3=displacement[[6]];
F1y=forces[[2]];
F2x=forces[[3]];
F2y=forces[[4]];
```

Results:

u_1=0.286·10^{-2} m=2.86 mm, u_3= 0.962·10^{-2} m=9.62 mm, v_3=-0.123·10^{-2} m=- 1.23 mm
A_y = -66666.67 N, B_x = -1.000·10^5 N, B_y = 66666.67 N

Definition of a scale factor

```
sf=5;
```

Definition of a polygon, variable: sf

```
Ldef=L+sf*Partition[displacement,2]
```

Display the deformed shape

```
Graphics[{
{White,EdgeForm[Thick],Polygon[Ldef]},
{Text[Style[„Deformed shape of TRUSS2D structure",15,Bold],{0.3,0.4}]}
}];
```

Animation of the deformed shape

```
Animate[
Graphics[{
{White,EdgeForm[Thick],Polygon[L+t*sf*Partition[displacement,2]]}
 },PlotRange->{{0,0.7},{0,0.5}}]
,{t,0,1},AnimationRunning->False]
```

Calculation of the beam diagrams, element displacement vector, force vector, stiffness matrix

Element 1

```
ue1={u1,v1,u2,v2};
Fe1=Ke1g.ue1;
```

Local force vector


```
Fe1L=T1t.Fe1;
```

Element 2

```
ue2={u2,v2,u3,v3};
Fe2ext={F1,0,0,0};
Fe2=Ke2g.ue2-Fe2ext;
Fe2L=T2t.Fe2;
```

Element 3

```
ue3={u1,v1,u3,v3};
Fe3ext:={0,0,0,0};
Fe3=Ke3g.ue3-Fe3ext;
Fe3L=T3t.Fe3;
```

Reduction factor (scale factor) for the plot of beam diagram

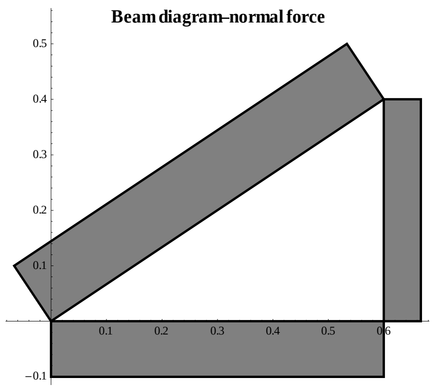
```
rf=1*10^-6;
```

Creation of the points of the beam diagram

```
Le1={{x1,y1},{x2,y2},{x2,y2+rf*Fe1[[3]]},{x1,y1-rf*Fe1[[1]]}};
Le2={{x2,y2},{x3,y3},{x3-rf*Fe2[[4]],y3},{x2+rf*Fe2[[2]],y2}};
Le3={{x1,y1},{x3,y3},{x3-rf*Fe3[[4]],y3+rf*Fe3[[3]]},{x1+rf*Fe3[[2]],y1-rf*Fe3[[1]]}};
```

Display the beam diagram by polygonplot

```
Graphics[{
{Gray, EdgeForm[Thick],Polygon[Le1]},
{Gray, EdgeForm[Thick],Polygon[Le2]},
{Gray, EdgeForm[Thick],Polygon[Le3]},
{Text[Style[„Beam diagram – normal force",15,Bold],{0.3,0.55}]}
},Axes->True
];
```



PROBLEM 5.2

Solve the problem using ANSYS or SIKER, verify the MATHEMATICA solution!